Classroom Discourse, Mathematical Rigor, and Student Reasoning: Analyzing the Dimensions of Powerful Mathematics Instruction and Learning

1.0 Overview

Low mathematics achievement in middle school—especially among poor and minority students—continues to be a national concern (National Assessment of Educational Progress, 2005; National Council of Teachers of Mathematics, 2000). Yet, even in the face of disappointing learning performances, state and national expectations for mathematics learning remain high. Despite often contentious "math wars" discussions, there is a broad consensus that students should master mathematics in a form that includes both skillful manipulation of symbols and procedures, and understanding of what these symbolic manipulations and procedures mean (Ball & Bass, 2000; Cobb, 2001; Gamoran, et al., 2003; Lampert, 1990; Porter, 2002; Secada et al., 1995;Wu, 1999).

Recognition of the need to integrate students' conceptual understanding, procedural competence and communicative abilities is supported by 30 years of cognitive science research. This research shows that accurate knowledge is foundational for learning with understanding. At the same time, it is clear that long-term retention of factual material is best secured when learners come to understand the logic and organization underlying the facts. Furthermore, the research shows that learning is most robust when learners become actively engaged in reasoning about the knowledge they are acquiring (Bransford et al., 2000; Resnick & Hall, 1998).

During the last two decades, a diverse group of researchers and educators have developed and implemented approaches to instruction that reflect this consensus. These integrated instructional approaches generally combine three dimensions of teaching: *intensive use of classroom discussion*, *mathematical depth and rigor in the curriculum and in its implementation,* and *attention to student reasoning*. Many studies have yielded promising outcomes, including significant improvements in student learning among low income and minority students.

1.1 Research on Instruction that Integrates Classroom Discourse, Mathematical Rigor, and Student Reasoning

Fifteen years ago, two of the present investigators (Resnick et al., 1992) demonstrated the potential of classroom discourse in primary grade mathematics instruction. Primary grade students were taught by a whole-class instruction method in which a problem was posed to the class and the teacher then led a structured discussion in which students explained why several different routes to solution were mathematically correct, and then conjectured and tested what would happen if, for example, quantities were changed or different geometric shapes were involved. Several cohorts of Bill's students, almost all lower income minority children, showed dramatic increases in computational skill and significant increases in comprehension and problem solving on standardized tests (Bill, et al., 1991).

Two other investigators in our group (Chapin & O'Connor, 2004), used two conceptually oriented curricula (*Investigations* and the *Connected Mathematics Project*) in a low-income urban school district to teach a mathematically demanding program to ethnolinguistic minority students. The goal was to identify and develop unrecognized talent and potential for "giftedness in mathematics." Like Bill's, the teaching in Chapin and O'Connor's "Project Challenge" classes called on students to engage in substantial teacher-guided discussion of the mathematics involved in the problems they worked on. Project Challenge teachers were supported to use a variety of academically productive “talk moves” and “talk formats,” designed to press students to explicate their reasoning and build on one another’s thinking. After two years in the program, the proportion of students rated as showing a "high probability of giftedness in mathematics" on the Test of Mathematical Abilities (TOMA) (Brown, Cronin & McEntire, 1994) rose from 4% to 41%. Results on the California Achievement Test (CAT) showed that Project Challenge students performed at high levels in both computation, and mathematical understanding and problem solving After three years in the project, 82% of each Project Challenge class on average scored “Advanced” or “Proficient” on the Massachusetts state assessment. The state average is 38%. Finally, in a post hoc, quasi-controlled comparison of students who had been eligible for Project Challenge (and matched with Project Challenge students), but not selected, the differences between Project Challenge students and their matched controls was significant and effect sizes were large (1.8). Building on this work, in an NSF ROLE grant, they were able to experimentally intensify the classroom discourse dimension and demonstrate links between particular kinds of teacher and student talk and student learning. Both studies showed effect sizes ranging from .8 to 1.3

There are a number of other such “success stories” in the literature on instructional change and school reform, where similar kinds of discourse-intensive instruction in difficult “traditional” high-demand subject matter have produced unexpected results. (See, among others, Lee, 2001, in literature; Ball & Lampert, 1998, Boaler, 2003, Boaler & Greeno, 2000, Chapin, et al., 2003, and Empson et al., 2006, in mathematics; Minstrell, 1989, in physics; Rosebery et al., 2005, Warren & Rosebery, 1996, in elementary science). Scholars in several European countries have reported similar results (e.g., Dooren, et al., 2005; Fischbein, Jehiam & Cohen, 1995; Merenluoto & Lehtinen, 2002; Tsamir, 2003; Van Vergnaud, 1989; Vosniadou, Baltas & Vamvakoussi, in press).

Opportunities for students to reflect and communicate about their mathematical work have been identified as essential for learning mathematics with understanding (Hiebert, et al., 1997), and effectively implementing high-level tasks. During a discussion, students can see how others approach a task and can gain insight into solution strategies and reasoning processes that they may not have considered. By engaging in whole-class, teacher-guided reflective discourse, students can explain their reasoning, make mathematical generalizations and connections between concepts, strategies or representations, and benefit from the collective mathematical work of the class for a given lesson or task (Cobb, et al., 1997; Lampert, 2001).

The effectiveness of discourse-intensive instruction depends significantly on the quality of the mathematical tasks used in instruction. A growing body of research indicates that tasks with high-level cognitive demands are important in improving students’ performance on state and national tests of mathematical achievement (e.g., Fuson et al., 2000; Riordan & Noyce, 2001; Schoen et al., 1999), in improving students’ understanding of important mathematical concepts (e.g., Ben-Chaim, et al., 1998; Huntley, et al., 2000; Reys et al., 2003; Thompson & Senk, 2001), and in improving students abilities to reason, communicate, problem-solve, and make mathematical connections (e.g., Blote et al., 2001; Ridgeway, et al., 2003; Schoenfeld, 2002; Torbeyns, Verschaffel & Ghesquiere, 2005; Torbeyns, J., Arnaud, L., Lemaire, P., & Verschaffel, L., 2004).

Mathematical tasks with high-level cognitive demands are characterized by multiple entry points and solution strategies, thereby allowing different students to approach the task in different ways, before being guided by the teacher into mathematically explicit formulations. High-level tasks can also feature multiple representations and opportunities to form connections between different mathematical ideas or representations (Hiebert et al., 1997; Stein, Grover & Henningsen, 1996; Stein & Lane, 1996). Tasks that are classified as having low levels of cognitive demand involve either memorization or the application of procedures with no connection to meaning or mathematical understanding (Doyle, 1983; Stein et al., 2000).

While the quality of tasks plays a significant role in learning, simply providing students with high-level tasks is insufficient for effective instruction. Research indicates that the level of cognitive demand of a task is often altered over the course of an instructional episode (Henningsen & Stein, 1997; Stein, Remillard & Smith, 2007). Teachers and students accustomed to traditional American styles of almost purely procedural teaching can be uncomfortable with the open discussion and intellectual struggle that often accompany high-level tasks (Clarke, 1997). Stein and Lane (1996) found, however, that the greatest student learning gains occurred in classrooms where students were consistently exposed to high-level tasks and in which the high-level cognitive demands were sustained throughout the lesson. These results appear consistent with findings from the TIMSS 1999 Video Study (Hiebert, et al., 2003), in which higher-performing countries were found to implement high-level tasks in ways that maintained the high-level cognitive demands. A set of factors that contribute to the decline of cognitive demands during classroom task implementation have been identified (Henningsen & Stein, 1997; Romanagno, 1994; Stein & Lane, 1996;). These factors are particularly strong in U.S. classrooms, according to the TIMSS 1999 Video Study, which showed less than 1% of classroom time in the U.S. is spent on high-level mathematical work. Similarly, in large-scale evaluative research in the U.S., Weiss and Pasley (2004), rated only 15% of the lessons as effectively supporting students' opportunities for learning mathematics.

There is evidence in a number of the studies cited that the mathematical demand of tasks often degrades during discussion and student learning suffers. Teachers may try to "help" students in ways that diminish the cognitive demand of the task (e.g., telling students which step to do next rather than helping them figure out what comes next and why). To keep the conversation moving and socially comfortable, discussions often devolve into teacher-led recitations, where teachers ask a question, a student answers and the teacher evaluates the answer, and then moves on to the next student, often referred to as the Initiation-Response-Evaluation, or IRE, sequence (Cazden, 2001).

1.2 Dimensions of Powerful Instruction

1.2.1 Classroom Discourse (CD). This dimension refers to the character of classroom interaction. As cognitive science has increasingly demonstrated the essential interplay between skilled performance and understanding in virtually all domains of knowledge, another line of research— blending linguistics and psychology—has emphasized the role of certain kinds of structured talk for learning with understanding (Anderson et al., 1997; Ball et al., 2003; Cazden, 2001; Chapin et al., 2003; Forman et al., 1998; Goldenberg, 1992/3; Heath, 1983; Lampert & Ball, 1998; Lee, 2001; Lemke, 1990; Mercer, 2002; Michaels & Sohmer, 2001; O'Connor, 2001; O'Connor & Michaels, 1996; Pontecorvo,1993; Resnick et al., 1993; Voss & Van Dyke, 2001; Warren & Rosebery, 1996; Wells, 2001; Wertsch, 1991; Yackel & Cobb, 1996.).

A number of researchers and educators have focused on the importance of language for minority students—as both a resource and as an obstacle in academic achievement generally (Adger et al., 2002; August & Hakuta, 1998; Ballenger, 1999; Baugh, 1999; Lee, 2001; Cazden, 2001; Delpit & Dowdy, 2002; Heath, 1983; Moll et al., 1992;Walqui & Koelsch, 2006) and more specifically in mathematics learning (Cocking & Mestre, 1988; Moschkovich, 2000; Moses &

Cobb, 2001). The classroom talk that is described in these interventions is markedly different from the standard recitation format of the traditional classroom.

The difficulty that teachers have in maintaining mathematical rigor and reasoning in their class discussions has led us to develop a system for training teachers in what we have come to call Accountable Talk strategies (Michaels, O'Connor, Hall & Resnick, 2002). Accountable Talk has three dimensions: Accountability to the community, Accountability to knowledge and Accountability to accepted standards of reasoning. The concept of Accountable Talk thus highlights the need to combine appropriate classroom discourse, mathematical rigor and student reasoning to achieve powerful mathematics instruction and learning.

The Accountable Talk form of classroom interaction is one in which the teacher poses a question that calls for a relatively elaborated response (in mathematics, both a solution and a reason for the solution) and then presses the class as a group to develop explanations for the solution. The process includes extended exchanges between teacher and student and among students, and includes a variety of talk moves, such as asking other students to explain what the first respondent has said, challenging students--sometimes via posing of counter examples, or "revoicing" a student's contribution ("So let me see if I've got your idea right. Are you saying…?"), which makes the student's idea, reformulated by the teacher, available to the entire group. A number of studies suggest that this kind of classroom discourse leads to deeper engagement in the content under discussion and surprisingly elaborated, subject-matter specific reasoning by students who might not normally be considered able students (e.g., Chapin & O'Connor, 2004; Cobb et al., 1996, 1997; Lampert, 2001; Lampert & Ball, 1998; Michaels, 2005; O'Connor, 1999, 2001; O'Connor & Michaels, 1996; Resnick & Nelson-LeGall, 1997; Rosebery, et al., 1992; Wells, 2001).

The six most important talk moves and an example of each move in its prototypical form follows: Talk Move (1) Revoicing: "So let me see if I've got your thinking right. You're saying XXX?" (with time for students to accept or reject the teacher's formulation); (2) Asking students to restate someone else's reasoning: "Can you repeat what he just said in your own words?"; (3) Asking students to apply their own reasoning to someone else's reasoning: "Do you agree or disagree and why?"; (4) Prompting students for further participation: "Would someone like to add on?"; (5) Asking students to explicate their reasoning: "Why do you think that?" or "How did you arrive at that answer?" or "Say more about that"; (6) Challenge or Counter Example: "Is this always true?" or "Can you think of any examples that would not work?"

In addition to a set of productive talk moves, teachers who use Accountable Talk engage students in a number of recurring *talk formats*, with stable norms for participation and turn-taking. Among these are: partner talk, whole group discussion, and small-group work. These talk moves and talk formats are extensively described in our Accountable Talk training CDs -- Michaels, et al., 2002, which will be one of the tools used in preparing teachers to lead structured mathematics discussion in our presently proposed study.

1.2.2 Mathematical Rigor (MR). Another dimension of powerful mathematics teaching is its greater rigor, density and focus on mathematics itself. This is a strong focus among mathematicians concerned with mathematics teaching (e.g. Parker & Baldridge, 2004; Wu, 1999) and others who believe that the mathematics itself is weak and/or thin in most American classrooms and that this is the major cause of low student achievement. Studies of teacher quality have shown that teachers who know more mathematics (typically as measured by holding an undergraduate degree in mathematics or certification in mathematics) generally have students who achieve at higher levels in mathematics (Darling-Hammond, 2000; Ferguson & Ladd, 1996; Hanushek, 1996; Rivkin, Hanushek & Kain, 2005), but these associations may result as much from current practices of assigning better prepared teachers to schools, or students within schools, who are of higher SES and are more tuned-in to school practices and demands.

The immediate task facing most schools and districts, especially in middle schools, is upgrading the mathematical rigor in classrooms of teachers already in place. This will require focusing instruction explicitly on the conceptual underpinnings of mathematics. Students who receive such instruction have been shown time and again to outperform those who do not (Brownell & Moser, 1949; Fuson and Briars, 1990; Hiebert & Grouws, 2007; Verschaffel, Greer & De Corte, 2007). The development of mathematical connections among ideas, facts and procedures leads to increased conceptual learning by students and also facilitates the development of important mathematical skills. Work by Rittle-Johnson and colleagues (2001) suggests that procedural and conceptual knowledge develops iteratively. The relationships between conceptual and procedural knowledge is bidirectional, and improved understanding of either type of knowledge can lead to improved understanding of the other type.

The Accountable Talk moves described above do not automatically carry mathematical rigor with them, even when used in conjunction with an appropriate high-cognitive-demand mathematics task. Teachers who do not have deep understanding of the mathematics embedded in the task problems, or whose mathematics knowledge does not extend beyond the specifics of what is in the textbook and teacher guidance materials, may accept student explanations and justifications that are incomplete or even incorrect. They may not see connections among concepts and procedures and revoice in ways that miss important opportunities to clarify and link mathematical ideas. And though they may "sprinkle" their queries to students with challenges and other forms of pressing for information, their challenges may not be crisply formulated in terms of the mathematics involved.

We might expect exactly this kind of instruction from many of the teachers currently teaching mathematics to students in urban school districts if the professional development they received focused only on strategies for managing classroom discourse. Many of today's middle school teachers do not have a strong preparation in mathematics. Only 41% of 8th grade students in 1999 received instruction from a teacher with formal training in mathematics including an undergraduate or graduate degree (National Center for Education Statistics, 2001). Middle school students continue to be taught mathematics by "out-of-field" teachers (Ingersoll, 2002), especially in urban schools. Many teachers struggle with the mathematics they are asked to teach and themselves lack the conceptual underpinnings as well as the procedural fluency that students are expected to master in grades 4–8 (Ball, 1990; Graeber, Tirosh & Glover, 1989; Ma, 1999; Post et al., 1992; Simon, 1993; Sowder et al., 1998; Stacey et al., 2001; Tsao, 2005). They may have a superficial grasp that enables them to follow the explicit steps outlined in a textbook, but this does not enable them to engage comfortably in classroom discourse in which students may formulate responses—especially initially—in unconventional ways, and they may be unable to steer the classroom conversation toward powerful mathematical formulations and generalizations, let alone discuss procedures clearly and accurately. Yet students' learning gains depend on just this kind of ability to sustain high levels of talk about the mathematical content. Ball and Bass (2003) found that teachers who were unable to choose helpful examples and interpret students' explanations confused their students and often were confused by their students. Hence, when teachers do not know the mathematics, their ability to identify the conceptual and pedagogical adequacy of students' explanations is limited. Furthermore, recent research by Hill, Rowan, and Ball (2005) found that teachers' knowledge of mathematics for teaching was related to student achievement. All of these findings make it clear that to effectively enact Accountable Talk practices, teachers must have a deeper understanding of the mathematics they are teaching, and, therefore, it will be necessary to specifically teach mathematics to teachers.

1.2.3 Student Reasoning (SR). Greater focus by teachers on mathematical depth and breadth may mean that *student* reasoning is largely overlooked by teachers, except when it closely matches the formally correct version of mathematical expression that the teacher is aiming to convey. Yet reports of virtually all of the integrated, discussion-based teaching we have described above stress that teachers continuously listen for and adapt to students' conceptions. Indeed, some scholars have argued that the most important focus for powerful mathematics teaching is student thinking about mathematics.

One important research program, the Cognitively Guided Instruction (CGI) project (Carpenter et al., 1999), worked on the assumption that, if teachers knew how children thought about basic mathematical operations, and if they actively looked for evidence of developmental sequences of understanding among their own students, they would be able to themselves develop a form of instruction that would be highly adaptive to student's thinking. A similar approach was used by the Integrated Mathematics Assessment (IMA) Program (Saxe, Gearhart & Nasir, 2001). Focusing on early mathematics instruction, these projects had an extensive body of research on children's mathematics thinking to draw on (e.g., Carpenter, Moser & Romberg, 1982; Leinhardt et al., 1992; Nunes, 1993), largely on topics such as addition, subtraction, multiplication and simple division of integers, that were intuitively available to children growing up in cultures that used counting and money systems based on decimals (Resnick, 1992). The developers of programs such as CGI and IMA provided extensive knowledge to teachers about this research, but did not directly provide teaching tasks, leaving it to teachers to devise the instruction. For the primary grades, where these intuitive bases for correct mathematics were prevalent, this worked very well, producing well-documented rises in student mathematics achievement in many classrooms.

It is not clear that this radical approach—educating teachers in intuitive mathematical thinking, but leaving the invention of specific tasks and pedagogical strategies to them—can be as successful for the whole of mathematics. Indeed the CGI group's efforts to extend the approach to higher levels in the curriculum produced much more limited learning gains than in the primary grades. This could be partly because, at the time CGI was operating, there was little research on students' development of math concepts beyond the most basic stages of number and arithmetic. There is more such research now. However, some of the newer research has also shown that student intuitions are as likely to produce mathematical misunderstandings as correct foundations for mathematics (e.g., Fischbein, Jehiam & Cohen, 1995; Merenluoto & Lehtinen, 2002; Tsamir, 2003; VanDooren et al., 2005; Vergnaud, 1989; Vosniadou & Verschaffel, 2004).

Nevertheless, most scholars agree that a very important aspect of teaching complex mathematics and scientific concepts is a continuous monitoring of student thinking by the teachers, and a conscious adaptation of instruction to students' understanding (Clark & Peterson, 1986; Shepard, 2001). Some (e.g., Hunt & Minstrell, 1994) have developed detailed diagnostic tools that can be used in the course of teaching, to make "on-line" diagnoses and respond immediately in the course of a classroom discussion to developing student understanding, including misconceptions (Shepard et al., 2005; Wiliam, 2007). This type of "formative assessment" can eventually lead to more robust learning, as students' misguided ideas are caught early on. In classrooms where thoughts are not made apparent, teachers cannot fully tap the depth of their students' understanding, or misunderstanding, until the end of a unit, or worse, until accountability measures are high-stakes testing situations. Many scholars have contributed information about how students' understanding of specific content develops (e.g., Battista, 2007; Carpenter, et al., 1989; Cobb, et al., 1991; Fuson et al., 1997; Hiebert & Wearne, 1993; Kieran, 2007; Lamon, 2007; Murray, Olivier & Human, 1994; Simon & Schifter, 1991), and what decisions teachers might make to facilitate student learning of that content for understanding. Hiebert, Carpenter, and colleagues (1997) developed a framework for analyzing features of classrooms that support students' understanding. Using this framework as a guide, teachers can examine lessons from their curriculum in terms of what they think is important to teach when stressing understanding, what they do when they teach for understanding and how they use a variety of pedagogical tools to support students' understanding (see also Stigler & Hiebert, 2004).