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Advanced Geometry Tutor: An Intelligent Tutoring System for Proof-Writing with Construction

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Abstract: Two problem solving strategies, forward chaining and backward chaining, were compared to see how they affect students' learning of geometry theorem proving with construction. In order to determine which strategy accelerates learning the most, an intelligent tutoring system, the Advanced Geometry Tutor, was developed that can teach either strategy while controlling all other instructional variables. Fifty-two (52) students were randomly assigned to one of the two strategies. Although computational modeling suggests an advantage for backwards chaining, especially on construction problems, the result shows that (1) the students who learned forward chaining showed better performance on proof-writing, especially on the proofs with construction, than those who learned backward chaining, and (2) the major reason for the difficulty in applying backward chaining appears to lie in the assertion of premises as unjustified propositions (i.e., subgoaling).

Keyword: Intelligent authoring system, Geometry Theorem Proving, Construction, Problem Solving Strategy

1. Introduction

Geometry theorem proving is known to be very challenging for students to learn [1]. The difficulty of the task skyrockets when it requires construction by compasses and a ruler as *a part of the proof*. Technically, one can add infinitely many segments and points to a problem figure, but until recently, no procedure was known for selecting an *appropriate* construction. We have hypothesized that if such a procedure were known, then teaching it should help students acquire this extremely challenging skill. We have further hypothesized that teaching a more computationally effective problem solving strategy might elicit faster learning.

For theorem proving that does not require construction, there are two common problem solving strategies: forward chaining and backward chaining. *Forward chaining* (FC for short) starts from given propositions and continuously applies postulates forwards. This continues until FC generates a proposition that matches the goal to be proved. *Backward chaining* (BC for short) starts from a goal to be proved and applies postulates backwards, that is, by matching a conclusion of the postulate to the goal, then posting the premises that are not yet proved as new goals to be proved.

In earlier work [2], we found a semi-complete algorithm for construction that is a natural extension of backward chaining, a common approach to proving theorems that do not involve construction. The basic idea is that a construction is done only if it is necessary for applying a postulate via backwards chaining. The same basic idea can be applied to the FC strategy.

We have conjectured that both BC and FC versions of the construction strategy are comprehensible enough for students to learn. A question then arises: would FC or BC better facilitate learning geometry theorem proving with construction?

To address the above issue, we have built an intelligent tutoring system, called *Advanced Geometry Tutor* (AGT) [3], and conducted a study to compare learning outcomes between students who learn FC and BC strategies.

2. Advanced Geometry Tutor

Figure 1 shows the graphical user interface of AGT. It consists of the five major components: (1) The *Problem Description window* shows a problem. The students can do construction upon the problem figure. (2) The *Proof window* shows a two column proof, which is a standard format taught in the American schools. (3) The *Message window*

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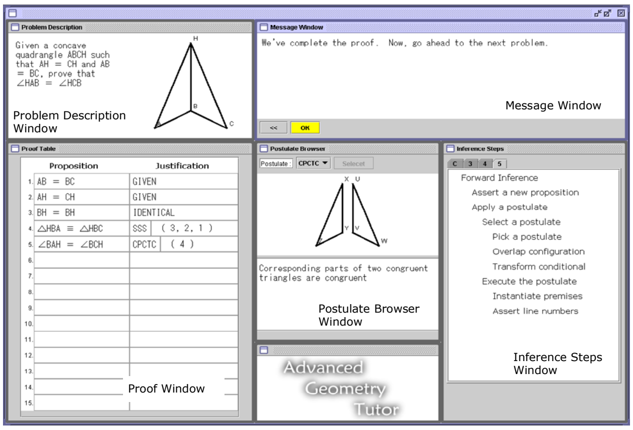

Figure 1: Advanced Geometry Tutor

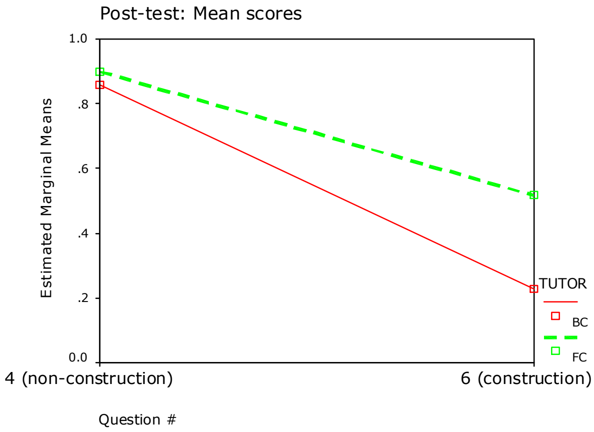

Figure 2: Mean scores on proof-writing problems in the post-test with and without construction

shows feedback from the tutor. This window is also used for the students' turn in a tutoring dialogue. (4) The *Postulate Browser* allows students to browse the postulates that are available for use in a proof. When the tutor provides scaffolding on how to apply a particular postulate to a particular proposition, the configuration of the postulate changes its shape so that the student can see how the postulate's configuration should be overlapped with the problem figure. (5) The *Inference Step window* displays the proof-writing procedure as a goal hierarchy of indented texts where each line corresponds to a single inference step in the procedure.

When students are solving a problem, the AGT provided both proactive and reactive scaffolding. Proactive scaffolding occurs before the step it addresses, whereas reactive scaffolding (feedback) occurs after the step.

3. Evaluation

52 graduate and undergraduate students were recruited for the study. The students were randomly assigned to one of the tutoring conditions where they used AGT individually to learn either the FC or the BC strategy.

Students studied a 9-page Geometry booklet, took a pre-test, used their version of AGT to solve 11 problems, and took a post-test. The pre- and post-tests were open book.

Figure 2 shows mean scores on the proof-writing questions in the post-test. The difference in the non-construction problem was not significant: $t(50) = 0.66$; $p = 0.51$, whereas the difference in the construction problem was significant: $t(50) = 2.89$; $p < 0.01$. That is, FC and BC students tied on non-construction problem, but FC students outperformed BC students on construction problem.

Analysis of how students wrote proofs revealed a significant difference in the use of premises between FC and BC students. That is, the BC students tended to leave the premises blank more often than the FC students. This tendency to leave the premise blank was one reason for the inferiority of BC students in writing correct proofs compared to the FC students.

4. Conclusion

The current study showed that proof-writing with construction can be taught with a technique that is a natural extension of theorem proving without construction. Although geometry construction is a difficult skill, it can be taught by contemporary ITS technology, given that the tutor has an explicit problem solving strategy to teach that will solve construction problems.

Despite the much higher computational demands of the FC version of the construction algorithm compared to the BC version, as documented in computational experiments with GRAMY, it turned out that FC students acquired more skill at construction than BC students. Our finding agreed with other empirical studies (e.g., [4]) showing novice students' difficulty in applying backward chaining.

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