Diagram Interaction During Intelligent Tutoring in Geometry: Support for Knowledge Retention and Deep Understanding

Kirsten R. Butcher (kbutcher@pitt.edu)
Learning Research & Development Center, University of Pittsburgh, 3939 O'Hara Street
Pittsburgh, PA 15260 USA

Vincent Aleven (aleven@cs.cmu.edu)
Human-Computer Interaction Institute, Carnegie Mellon University, 5000 Forbes Avenue
Pittsburgh, PA 15213 USA

Abstract

Prior research has shown that skilled problem solvers often use features of visual representations to cue relevant knowledge, but little is known about how to support learners in developing connections between visual and verbal knowledge components. In this research, we investigated two methods to support focus on key visual features during problem solving in an intelligent tutor: 1) student interaction with diagrams during problem solving, and 2) student explanations that connected diagram features to geometry rules at each problem-solving step. Research was conducted in 10th grade classrooms using an experimental version of the Geometry Cognitive Tutor. Interaction with diagrams promoted long-term retention of problem-solving skills and supported deep understanding of geometry rules, as evidenced by items testing transfer and visual-verbal knowledge integration. Diagram-rule explanations did not significantly influence learning. Findings suggest that student focus on relevant visual information should be carefully integrated into problem-solving practice to support robust learning.

Keywords: visual interaction; diagrams; geometry; learning; transfer; retention; intelligent tutoring

Visual Representations in Skilled Performance

Existing research has found that experts use visual representations in rich and interconnected ways during skilled problem solving. Stylianou (2002) studied the problem-solving processes of professional mathematicians and noted that mathematicians used diagrams extensively to inform their analysis of the problem, their selection of subgoals, and their eventual solutions. During problem solving, mathematicians created visual representations in a step-by-step manner, where visual information in the representation was analyzed at each step in order to inform reasoning and to cue relevant approaches. Mathematicians recognized important features and patterns in their diagrams and revised or annotated their diagrams to reflect the outcome of their analysis at each step.

Stylianou's (2002) results complement previous research in expert problem solving that has demonstrated close connections between visual representations and existing knowledge. Koedinger and Anderson (1990) found that experts solving geometry problems made inferences that were strongly tied to geometry diagrams, and that features in the problem diagrams cued relevant problem-solving steps. Koedinger and Anderson found that the problem solving steps mentioned and skipped by experts could be successfully predicted by a model (the Diagram Configuration Model) that parsed diagrams into key geometric configurations and used these configurations to cue relevant schemas. The development of skilled performance in geometry appears to be correlated with attention to key diagram features, as well as successful association of those features with relevant geometry rules.

Recent eye-tracking research suggests that learner focus on key visual information predicts successful performance even among non-experts. On insight problems, success is correlated with greater attention to the problem elements that must be manipulated for a solution (Knoblich, Ohlsson, & Raney, 2001). Unlike unsuccessful solvers, successful solvers fixate on key elements more frequently and increasingly devote long fixations to these key elements. Eye-tracking results from comprehension research have also suggested the importance of learner focus on key visual features: deep comprehenders fixate on critical features of a visual representation just before or during the generation of deep-reasoning questions concerning those features (Graesser, Lu, Olde, Cooper-Pye, & Whitten, 2005).

Focusing student attention on key visual features that are relevant to problem solving should help students learn to use visual representations in domain-meaningful ways. Research suggests that novices do, in fact, attend to visual representations during learning but that they often use visual features in a superficial way that does not support successful analysis. Lowe (1993; 1999) found that novices often are unable to discriminate between relevant and irrelevant aspects of science visualizations, and are easily distracted by domain-irrelevant but perceptually-salient features. In geometry, Lovett and Anderson (1994) found that diagrams, not problem logic, form the basis of novice memories for geometry proof. Geometry problems that used the same diagram but required different underlying solutions were solved significantly less often than when the problems with differing logic were accompanied by different diagrams. Thus, superficial and unfocused attention to diagrammatic information can compromise problem-solving performance.

Existing research with experts and successful learners, then, suggests that students may benefit from a learning environment that helps them to focus their attention on key visual features as they learn to solve problems. Increasing student attention to key visual features that are involved in problem-solving steps may support the development of connections between relevant visual and verbal knowledge, resulting in deeper understanding and more successful problem solving with visual representations.

Objectives of the Study

In this study, we examined the effectiveness of two methods for connecting key diagram elements to geometry knowledge: 1) the use of interactive visual diagrams during problem solving, and 2) the use of specific explanations stating which diagram features were relevant to the geometry rules that students used during problem solving. We specifically were interested in effects on practiced problem solving, transfer items testing deep understanding of geometry rules, integration of visual-verbal knowledge, and long-term retention of problem-solving skills.

Method

Participants

Participants were 104 students in 10th grade Geometry at a rural Pennsylvania school. At posttest, we excluded students who had been absent for at least one of the study sessions or at posttest, leaving 81 students for analysis. An additional 23 students were absent at the delayed posttest, leaving 58 students for analyses. The timing of this study corresponded to the placement of the curriculum in the class schedule; the relatively high attrition at delayed posttest is likely due to a high absentee rate in the first school week following winter vacation when the delayed posttest was given. Attrition was comparable across conditions, as seen in Table 1.

The Geometry Cognitive Tutor

The current work used the Cognitive Tutor as a research vehicle. The Cognitive Tutor is an intelligent tutoring system built upon the ACT-R theory of learning and cognition (Anderson & Lebière, 1998) that has been used successfully to support student learning in algebra (Anderson, Corbett, Koedinger, & Pelletier, 1995; Koedinger, Anderson, Hadley, & Mark, 1997) and geometry (Aleven & Koedinger, 2002). Cognitive Tutors support student learning by doing; the Tutor selects problems for students to complete during practice, forming a model of students' competencies based on their success with skills contained in those problems. At every problem-solving step, the Cognitive Tutor provides feedback on student responses, gives hints upon request or after repeated errors, and tracks students' skill development. Other publications (e.g., Aleven & Koedinger, 2002; Anderson et al., 1995; Corbett, McLaughlin, & Scarpinatto, 2000) have described development details of the Cognitive Tutor. For the purposes of this research, we did not modify the ways in which the Geometry Cognitive Tutor modeled student knowledge, nor the content of the Cognitive Tutor curriculum (problems, hints, etc.). As seen in Table 1, we varied the site of student interaction with the Tutor (a textual solution table vs. the visual diagram of the problem) and the type of explanation required for problem-solving steps (whether students state geometry rules only vs. the geometry rule *and* its application to the diagram).

Table 1: Experimental conditions, with sample sizes at posttest and delayed posttest assessment.

		Site of Interaction	
		Table	Diagram
Type of Explanation	Geometry Rule Only	Posttest: n = 17 Delayed Posttest: n =12 *Attrition n = 14*	Posttest n = 20 Delayed Posttest: n =15 *Attrition n = 9*
	Geometry Rule & Diagram Application	Posttest n = 22 Delayed Posttest: n =15 *Attrition n = 11*	Posttest n = 22 Delayed Posttest: n =16 *Attrition n = 11*

The first experimental factor—the site of interaction—varied the interface location where students interacted with the Tutor. In the diagram interaction condition, students clicked on question marks that appeared at the location of key diagram features related to each problem-solving step. Clicking a question mark opens a nearby work area where students enter answers and explanations. Once a numerical answer is accepted, it is displayed in place of the question mark at the appropriate diagram location. The diagram interaction condition can be seen in Figure 1; in this example, the student already has solved Arc EO and now is entering the answer and explanations for Angle OTE in the work area visible in the bottom left corner of the screen.

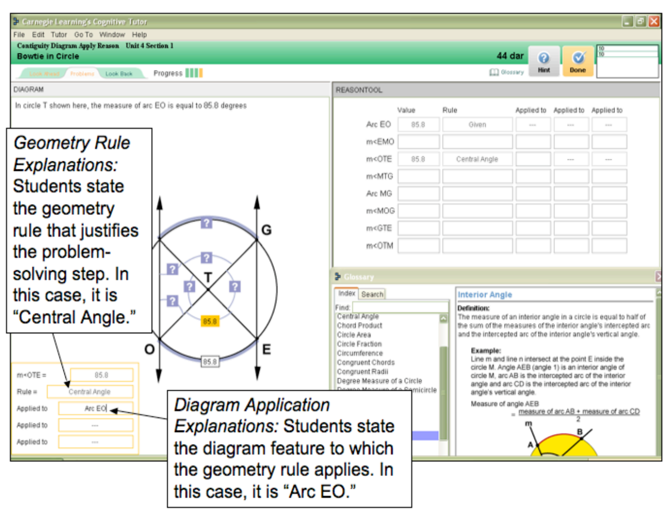

Figure 1: Student explanations of problem-solving steps, as implemented in the diagram interaction condition.

In the table interaction condition, the diagram is visible but not interactive and the diagram does not change during problem solving. Students can see question marks in the diagram that appear for key diagram features, but they cannot click on these icons and accepted answers do not appear in the diagram. All answers and explanations are entered as text in the solution table seen in the upper right corner of Figure 1, and accepted answers are displayed in this table. In the diagram condition, the table is visible but it is not interactive.

The second experimental factor – type of explanation – varied the explanations required to justify a problem-solving step. Explanations in the Geometry Cognitive Tutor are implemented as menu-based selections. Previous research has shown that menu-based explanations are successful in supporting student learning (Aleven & Koedinger, 2002). *Geometry rule* explanations required students to justify each problem-solving step by selecting the geometry rule (e.g., “central angle”) that they had used to calculate their answer. *Diagram application* explanations additionally required students to select the known diagram element(s) to which the selected geometry rule applied (e.g., “Arc EO”). Figure 1 shows a screen shot of the experimental interface with explanations relevant to the depicted problem.

Assessments

Posttest The posttest assessment included 8 problem-solving situations with two questions per problem, for a total of 16 questions. As seen in Figure 2, each assessment problem contained four types of items: problem-solving items as practiced in the Cognitive Tutor (numerical answers and geometry rules), transfer items requiring troubleshooting (solvability decisions), and visual-verbal integration items (diagram application).

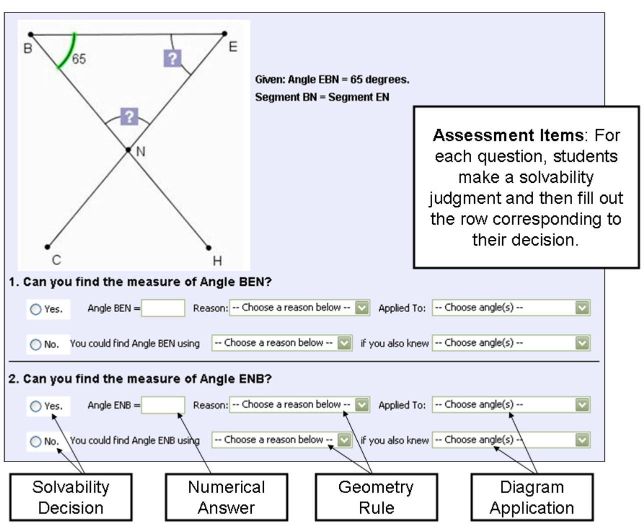

Figure 2: Example assessment problem.

Each problem presented a diagram accompanied by given information (see Figure 2), such as the measure of one or more angles and information about geometry relationships present in the diagram (e.g., parallel lines or equal line segments). The solvability decision was prompted by questions that asked whether there was enough information to find a specific diagram element, “Can you find the measure of Angle BEN?” If students selected “Yes,” they then solved the problem by finding the angle’s numerical measure, indicating the geometry rule they used to find their answer, and selecting the known diagram element(s) that allowed them to use their selected geometry rule. If students selected “No” in response to the solvability decision, they indicated a geometry rule that *could* be used to solve the problem and the additional diagram information that would be needed in order to use that rule.

Delayed Posttest The delayed posttest included four problem solving situations with two questions per problem, for a total of eight questions. Questions were isomorphic to problems on the posttest.

Tutor Log Data Every student transaction with the Geometry Cognitive Tutor is recorded in a log file that includes information about the problem and step being attempted, the type of student action (e.g., hint request, answer input), and a time stamp. These data allow indirect assessment of student processes during learning. In this research, we analyzed the number of problems that students completed during practice, and the amount of time taken to enter answers and to state geometry reasons during practice.

Procedure

Students participated in the study as part of their normal classroom curriculum, in which the Geometry Cognitive Tutor is used to augment teacher-led lessons and activities. Students practice geometry problem solving using the Cognitive Tutor for one classroom block (approximately 1 hour 15 minutes) each week. All students spent three classroom blocks working in the angles units of our experimental version of the Geometry Cognitive Tutor, taking a posttest in the fourth week. One month following the posttest, students completed a delayed posttest. Students were instructed to do their best to answer every question, and to take a guess if they were unsure of an answer.

The study was conducted as a 2 X 2 design. Grade-matched quartets of students were randomly assigned to the four experimental conditions within each of the participating classrooms. All classes were taught by the same teacher. The classroom teacher was instructed to interact with students as usual, providing typical levels of help and support during intelligent tutoring practice.

Results

Student performance at posttest and Geometry Cognitive Tutor log data were analyzed using a multiple analysis of covariance (MANCOVA) on the 81 students with complete data at posttest. Student progress in the Tutor prior to the start of the study was used as the covariate. Prior progress in the Tutor is a useful covariate because it reflects both student grades and prior knowledge. In the studied classrooms, students' Cognitive Tutor progress is used to calculate a significant portion of their class grade and, since progress in the Cognitive Tutor is self-paced, Tutor progress gives a more accurate indication of exposure to geometry concepts than the concepts covered in class.

Student performance at delayed posttest was also analyzed by a MANCOVA with prior Cognitive Tutor progress as the covariate, using data from the 58 students who had complete data at delayed posttest.

Posttest Performance

Practiced Skills At posttest, success with problem-solving skills practiced in the Geometry Cognitive Tutor (finding numerical answers and stating geometry rules) were not different for any experimental condition. Neither diagram interaction nor explaining the connection between geometry rules and diagrams during intelligent tutoring practice led to significantly better problem solving or rule-based explanations in the short term. Given extended student practice in an already-successful intelligent tutor that supports mastery learning, it is not particularly surprising that students perform similarly on these practiced items immediately after tutoring.

Transfer Items Solvability decisions were required for every question at posttest, but were not practiced during tutoring by any experimental condition. In the Geometry Cognitive Tutor, all problems can be solved and this may lead students to apply shallow strategies during practice. For example, students may calculate an answer using a common formula (e.g., subtract a given value from 90 or 180) and enter a recently-used rule without deeply understanding its application to the current problem. Solvability decisions, in contrast, require students to analyze both the problem diagram and the given information to determine if there is a geometry rule that can be appropriately applied to the problem situation.

As reflected in the means in Table 2, at posttest students who had interacted with diagrams tended to outperform students who had worked with tables in detecting solvable and unsolvable problems ($F_{(1,\ 76)} = 2.70$, $p = .10$, $\eta_p^2 = .034$). The type of explanation completed did not affect solvability decisions ($F < 1$).

Visual-Verbal Integration Diagram application items were used to assess the integration of visual-verbal knowledge by evaluating connections between key visual features and relevant geometry rules. These items require students to indicate the diagram features (typically one or two angles) used in the application of the geometry rule selected in their answer. Experimental conditions differed in whether or not they had practiced this skill in the tutor. Students in the *geometry rule and diagram application* explanation condition did practice this skill during tutoring; they selected a geometry rule and the relevant diagram features at every problem-solving step during practice and the tutor provided feedback for these explanations. However, students in the *geometry rule only* explanation condition received no practice or feedback in selecting diagram elements relevant to geometry rules. For students in the *geometry rule only* explanation condition, diagram application items also test deep transfer that relies upon integrated visual-verbal knowledge.

As seen in Figure 3, students in the diagram interaction conditions performed better on diagram application items than students in the table interaction conditions ($F_{(1,\ 76)} = 6.23$, $p = .02$, $\eta_p^2 = .076$). This effect is driven by student performance in the *geometry rule only* explanation condition. Separate analysis of the *geometry rule only* explanation condition shows that diagram interaction is powerful in supporting connections between visual representations and domain knowledge, with students in the diagram interaction condition demonstrating significantly greater success in linking key diagram features to relevant geometry rules ($F_{(1,\ 34)} = 11.51$, $p = .002$, $\eta_p^2 = .253$).

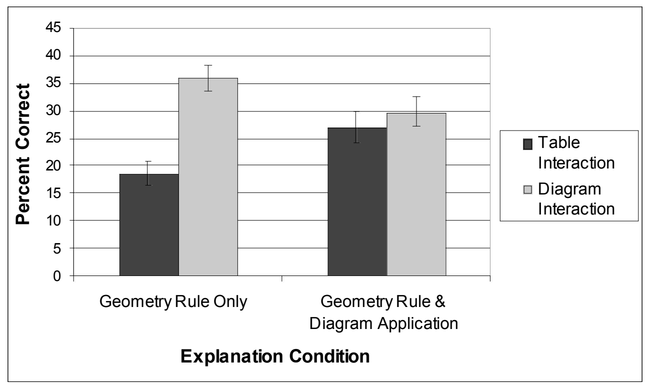

Figure 3: Mean (+ SE) performance on diagram application items at posttest.

Interaction with diagrams provides implicit but powerful support for making domain-relevant connections between visual and verbal information sources. As seen in Figure 3, students who interact with the table rather than the diagram may get some benefit from the diagram application explanations. However, diagram application explanations do not have additive benefits beyond what is achieved by interaction with visual diagrams.

Delayed Posttest Performance

Practiced Skills At delayed posttest, students who worked with the interactive diagrams showed a significant advantage in the accuracy of their problem-solving answers compared to students who worked with the interactive tables ($F_{(1,\ 53)} = 4.03$, $p = .05$, $\eta_p^2 = .071$). In contrast, the type of explanation produced during practice had no effect on long-term retention of problem-solving skills. As seen in Figure 4, diagram interaction supported successful problem solving as measured by correct numerical answers for geometry problems.

Table 2: Mean and (*standard deviation*) for percent correct on assessment items and tutor log data.

	Table Interaction		**Diagram** Interaction	
	Explain **Geometry Rule**	Explain **Geometry Rule & Diagram Application**	Explain **Geometry Rule**	Explain **Geometry Rule & Diagram Application**
Posttest Solvability Decisions	59.2% (*20.1*)	56.0% (*25.0*)	65.6% (*21.3*)	65.1% (*22.4*)
Posttest Numerical Answers	35.8% (*26.0*)	31.8% (*34.7*)	43.8% (*33.9*)	37.9% (*28.6*)
Posttest Geometry Rules	20.2% (*16.6*)	22.4% (*24.2*)	25.6% (*21.3*)	22.7% (*25.8*)
Posttest Diagram Application	18.8% (*18.1*)	27.0% (*25.4*)	35.9% (*20.9*)	29.8% (*25.6*)
Delayed Posttest Solvability Decisions	56.3% (*18.8*)	47.5% (*15.8*)	57.5% (*22.6*)	63.3% (*16.8*)
Delayed Posttest Numerical Answers	20.0% (*27.0*)	24.0% (*28.5*)	36.0% (*32.3*)	36.3% (*29.4*)
Delayed Posttest Geometry Rules	14.6% (*18.3*)	12.5% (*15.7*)	15.8% (*22.9*)	25.8% (*28.3*)
Delayed Posttest Diagram Application	14.6% (*17.5*)	21.7% (*17.3*)	23.3% (*21.6*)	23.4% (*25.0*)
# Problems Completed (overall M =	59.5 (19.5)	61.45 (25.5)	61.20 (19.0)	55.14 (21.4)
Average Time (sec): Answers	28.8 (*16.0*)	27.4 (*15.5*)	26.2(*13.0*)	24.0 (*12.0*)
Average Time (sec): Geometry Rules	9.3 (*3.5*)	10.3 (*7.8*)	9.4 (*2.9*)	10.7 (*6.6*)

Overall, the problem-solving success for students who interacted with the textual solution tables declined by an average of nearly 35% in the month following practice, compared to an almost 12% average decline for students who interacted with the visual diagrams during practice. The pattern of means for geometry rule responses also favors students who interacted with diagrams during problem-solving practice in the Tutor (see Table 2), but this effect is not statistically significant ($F_{(1, 53)} = 1.79$, $p = .19$)

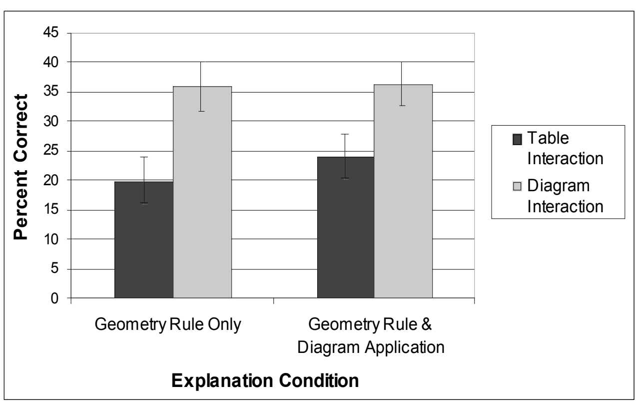

Figure 4: Mean (+ SE) performance on numerical answers at delayed posttest.

Transfer Items and Visual-Verbal Integration At delayed posttest, students who used the interactive diagrams tended to make more accurate solvability decisions than students who interacted with the textual tables during practice ($F_{(1, 53)} = 3.11$, $p = .08$, $\eta_p^2 = .055$). Explanations did not influence solvability decisions at delayed posttest ($F < 1$). For diagram application items, no significant condition differences were found at delayed posttest.

Process Analyses Using Tutor Log Data

The current results show that diagram interaction can support integration of visual-verbal knowledge, transfer performance, and long-term knowledge retention of problem-solving skills. However, the learning outcomes themselves do not address whether these benefits resulted from increased focus on key visual features during use of the interactive diagrams or from other process differences resulting from the use of interactive diagrams. One might argue that interactive diagrams supported learning because they freed students from the effort needed to switch attention or translate between the table and diagram. If this were the case, it is likely that students using interactive diagrams would complete more problems in the Geometry Cognitive Tutor and/or enter their answers more quickly.

Analyses of student transactions recorded in the Tutor log data showed no evidence that the interactive diagrams changed students' pattern of responses in the Tutor nor hastened their progress through the problems. Students who interacted with the tables and diagrams did not differ in the time that they took to enter answers ($F_{(1, 76)} = 1.04$, $p = .31$) or reasons ($F < 1$) for problem-solving steps. Students also did not differ in the number of problems that they were able to complete during use of the Tutor ($F < 1$). Our interpretation of these results is that the interactive diagrams change the focus of student attention in domain-relevant ways during typical problem-solving activities. However, additional research is necessary to determine the precise support offered by interactive diagrams and subsequent effects on student learning processes.

Conclusions

Results showed diagram interaction to have surprisingly robust benefits for learning. Interactions with geometry diagrams during intelligent tutoring supported the long-term retention of practiced problem-solving skills, promoted the development of integrated visual-verbal knowledge, and supported student performance on transfer items that required students to use visual representations to analyze the applicability of conceptual knowledge. The addition of explanations that explicitly connected diagram elements to geometry rules did not demonstrate strong effects on learning, showing generally weaker benefits than diagram interaction.

Why didn't the addition of explanations that focused on diagram elements have more impact? One possibility may be that diagram application explanations typically were the last action performed on each problem-solving step and may have been largely redundant with processing that occurred earlier (in determining the numerical answer and geometry rule for the same step). In fact, on average, students' diagram application explanations were correct 88% of the time. More meaningful connection of visual representations may occur as students struggle to find appropriate solutions to problems, making it necessary to embed visual interaction at these earlier, critical times. It is also possible that the diagram application explanations did not prompt students to attend to relevant diagram features in the way we anticipated. Students may have focused not on diagram configurations, but on the numerical values in the diagram (or table) that they used in their calculation of the numerical answer for the same step. In current work, we are exploring new forms of diagram interaction at key opportunities for learning, by supporting student highlighting of relevant diagram elements following errors. We also are exploring problem-solving processes associated with student development of visual-verbal connections via think-aloud protocols collected from students interacting with tables or diagrams during intelligent tutoring. Future work should continue to explore the rationale for visual interaction benefits, with particular focus on specifying the processes involved in visual-verbal coordination during learning.

Acknowledgments

The authors thank Octav Popescu, Andy Tzou, Carl Angiolillo, Michael Nugent, Thomas Bolster, and Grace Leonard for their contributions. Special thanks to Colleen Conko for her help and dedication to this research. This work is supported by the Pittsburgh Science of Learning Center through funding from the National Science Foundation (#0354420). Opinions and conclusions presented here are those of the authors and do not necessarily reflect the funding agency.

References

Aleven, V., & Koedinger, K. R. (2002). An effective metacognitive strategy: Learning by doing and explaining with a computer-based Cognitive Tutor. *Cognitive Science, 26*(2), 147-179.

Anderson, J. R., Corbett, A. T., Koedinger, K. R., & Pelletier, R. (1995). Cognitive tutors: Lessons learned. *Journal of the Learning Sciences, 4*(2), 167-207.

Anderson, J. R., & Lebière, C. (1998). *The Atomic Components of Thought*. Mahwah, NJ: Erlbaum.

Bodemer, D., Ploetzner, R., Feuerlein, I., & Spada, H. (2004). The active integration of information during learning with dynamic and interactive visualisations. *Learning and Instruction, 14*, 325-341.

Corbett, A. T., McLaughlin, M., & Scarpinatto, K. C. (2000). Modeling student knowledge: Cognitive tutors in high school and college. *User Modeling and User-adapted Interaction, 10*(2/3), 81-108.

Graesser, A. C., Lu, S., Olde, B. A., Cooper-Pye, E., & Whitten, S. (2005). Question asking and eye tracking during cognitive disequilibrium: Comprehending illustrated texts on devices when the devices break down *Memory & Cognition, 33*(7), 1235-1247.

Kalyuga, S., Chandler, P., & Sweller, J. (1999). Managing split-attention and redundancy in multimedia instruction. *Applied Cognitive Psychology, 13*(4), 351-371

Knoblich, G., Ohlsson, S., & Raney, G. E. (2001). An eye movement study of insight problem solving. *Memory & Cognition, 29*(7), 1000-1009.

Koedinger, K. R., & Anderson, J. R. (1990). Abstract planning and perceptual chunks: Elements of expertise in geometry. *Cognitive Science, 14*(4), 511-550.

Koedinger, K. R., Anderson, J. R., Hadley, W. H., & Mark, M. A. (1997). Intelligent tutoring goes to school in the big city. *International Journal of Artificial Intelligence in Education, 8*, 30-43.

Kozma, R. (2003). The material features of multiple representations and their cognitive and social affordances for science understanding. *Learning and Instruction, 13*, 205-226.

Lovett, M. C., & Anderson, J. R. (1994). Effects of solving related proofs on memory and transfer in geometry problem solving. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 20*(2), 366-378.

Lowe, R. K. (1993). Constructing a mental representation from an abstract technical diagram. *Learning & Instruction, 3*, 157-179.

Lowe, R. K. (1999). Extracting information from an animation during complex visual learning. *European Journal of Psychology of Education, 14*(2), 225-244.

Stylianou, D. A. (2002). On the interaction of visualization and analysis: the negotiation of a visual representation in expert problem solving. *Journal of Mathematical Behavior, 21*, 303-317.

Tarmizi, R. A., & Sweller, J. (1988). Guidance during mathematical problem solving. *Journal of Educational Psychology, 80*(4), 424-436.