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Cue recognition and cue elaboration in learning from examples

XINMING ZHU[†], YIFEI LEE[†], HERBERT A. SIMON[‡], AND DAN ZHU[§]

[†]Chinese Academy of Sciences, Beijing, People's Republic of China; [‡]Carnegie Mellon University, Pittsburgh, PA 15213; and [§]University of Iowa, Iowa City, IA 52242

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ABSTRACT **This paper describes the processes used by students to learn from worked-out examples and by working through problems. Evidence is derived from protocols of students learning secondary school mathematics and physics. The students acquired knowledge from the examples in the form of productions (condition → action): first discovering conditions under which the actions are appropriate and then elaborating the conditions to enhance efficiency. Students devoted most of their attention to the condition side of the productions. Subsequently, they generalized the productions for broader application and acquired specialized productions for special problem classes.**

Introduction

The effectiveness of instructional methods in which students learn from worked-out examples and by solving problems [learning from examples and by doing (LFED)] has been demonstrated in several contexts (1–5). While LFED has a sound theoretical foundation (6, 7), data are still scanty on the sequences of events that lead students using these methods toward mastery of skills. In this paper, we illustrate the central importance for skill acquisition of the condition sides of the productions that cue appropriate problem solving actions.

Method. In the experiments that produced the data discussed here, the control groups were taught in the traditional way. In the experimental groups, the teacher did not lecture but, instead, students worked individually through the study materials. In this paper, we will not assess the general effectiveness of the LFED methods but will examine some protocols of individual subjects in a physics task that reveal the critical role of the condition sides of productions for skilled performance.

While studying the examples, the subjects had available the correct answers, so that they could get feedback at any time. Typically, they first tried to solve the problems and then checked with the correct answer.

Production Systems. According to current cognitive theories, the knowledge for skilled performance is stored in productions: if-then statements consisting of a set of conditions (C) followed by a set of actions (A), C → A. Whenever the conditions of a production are satisfied, the action is carried out. The cognitive theories predict, with good supporting evidence, that students can learn, using LFED, the productions they need for effective performance.

A simple example of a production is *If the goal is to add a column of figures, then hold in memory the cumulative total, set it initially to 0, and add to it each of the successive figures, from top to bottom.*

IF Goal[Add-column(x)],
THEN Set cumulative-total = 0;
Add-next-item-to(cumulative total);
advance pointer; repeat to end of column.

A person who has this production and this goal (the condition) will place 0 in memory, add the first number (say 3) and replace the 0 by 3, add the next (say 6), and replace the 3 by 9, and so on, until the list is exhausted and the cumulative total can be reported.

The student may or may not be able to verbalize the productions used. If a production is in memory, (*i*) whenever one of its conditions is in the focus of attention, it will be noticed, and (*ii*) when all the conditions are satisfied, the action will be carried out. Productions represent skill, not declarative knowledge.

An adaptive production system (APS) learns by modifying itself—altering productions and adding others to memory. An early APS learned to solve linear equations in algebra (6). At about the same time, humans were modeled as APSs having domain-specific knowledge (7).

Zhu (4) and Zhu and Simon (5) demonstrated, through extensive instructional experimentation in schools, that LFED is both effective and efficient in terms of learning time. Anderson and his colleagues (8, 9), also using production system models of students' skills, have constructed effective computer tutors for geometry, algebra, and LISP programming and have tested them in schools.

Learning the Conditions of Productions. A student needs to know not only what actions can be performed (the laws of the domain) and how to perform them but also under what conditions each action is appropriate. While traditional classroom instruction and textbooks have emphasized domain laws (actions), experience with LFED indicates that acquiring appropriate conditions is the largest learning task.

For example, in algebra, students are taught that they may add the same quantity to both sides of an equation (or subtract, multiply, or divide on both sides) without altering the values of the unknowns. This does not explain *when* to apply each action to solve an equation. For that, the learner must learn conditions for each action. For example, the following productions can solve many linear equations.

P1: If there is a numerical term on the left side of the equation
→ subtract it from both sides.

P2: If there is a term in x on the right side of the equation
→ subtract it from both sides.

P3: If there is a term in x on the left side of the equation whose coefficient is not unity
→ divide both sides by the coefficient.

Thus,

$$7x + 8 = 3x + 24 \rightarrow \text{(Apply P1 and collect terms.)}$$
$$7x = 3x + 16 \rightarrow \text{(Apply P2 and collect terms.)}$$
$$4x = 16 \rightarrow \text{(Apply P3.)}$$
$$x = 4.$$

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Abbreviation: LFED, learning from examples and by doing.

Contrary to the emphasis in typical textbooks, learners using LFED methods direct most of their attention to conditions and the associations of conditions with actions. In the example, the 8 on the left side signals the appropriateness of applying P1, the $3x$ on the right signals P2, the 4 on the left signals P3. If the actions (or laws) are stressed while ignoring the conditions, learners will be unable to apply the laws.

While an instructor develops a proof on the blackboard, the students may check each step; but at the end, many students wonder "How did the instructor happen to choose the right sequence of steps?" In LFED, examples and problems are selected to teach both the actions and the conditions under which particular actions should be taken.

Learning About Buoyancy

We turn now to examples of protocol material from learners who were applying LFED to the study of buoyancy.

Material. Buoyant force is the net upward force exerted on a floating or submerged body. Productions compare the magnitudes of buoyant forces on various bodies. Students were asked to examine examples and solve problems and, to enhance their awareness of the conditions for taking actions, were asked to explain their actions. Every problem was followed by a sequence of subproblems designed to call attention to the relevant conditions and their cues.

Buoyancy. All buoyancy problems fall under Archimedes' rule but may be solved in simpler ways if special cases can be recognized. Table 1 defines terms we will use and lists some useful productions. Archimedes' rule states *The buoyant force exerted by a liquid upon an object equals the weight of liquid displaced by the object.* Formally (see Table 1),

$$\text{in}(x,y) \rightarrow \text{Fb}(x,y) = \text{Vim}(x,y) \times \text{Den}(y)$$

The weight of liquid displaced is, by definition, the weight of a volume of liquid just equal to the volume, Vim(x,y), of the portion of the object below the surface of the liquid. The conclusions derivable from the rule depend on whether an object floats on the liquid or is totally submerged in it.

(*i*) If the object *floats*, the buoyant force equals the object's *weight*; otherwise it would sink deeper (see P1 and P2; Table 1).

(*ii*) If the object *is wholly submerged*, the buoyant force depends solely on its *volume* (equal to the weight of liquid that fills this volume and less than the weight of the object) (see P3, P4; Table 1).

Many productions corresponding to easily recognizable situations can be learned once and for all and used without returning each time to Archimedes' law. Below we have italicized the conditions under which each production acts. These productions are written formally in Table 1.

PA. If two or more objects have the *same volume* and are all *fully submerged* in a liquid,
→ the buoyant forces on all the objects are equal.

PB. If two or more objects have the *same weight* and all *float* on a liquid,
→ the buoyant forces on all the objects are equal.

PC. If several objects all *float* on a liquid,
→ the greater an object's weight, the greater the buoyant force on it.

PD. If several objects are all *fully submerged* in a liquid,
→ the greater an object's volume, the greater the buoyant force on it.

PE. If an object is *first submerged in one liquid, then in another, denser than the first*,
→ the buoyant force exerted by the denser liquid will be greater than that exerted by the less dense.

Table 1. Some productions for buoyancy

Notation
Fb(x)—Buoyant force on x
Den(x)—Density of x
Vol(x)—Volume of x
Wt(x)—Weight of x
Wt(x) = Den(x) × Vol(x)
Net(x) = Wt(x) − Fb(x)—Net weight of x
Vim(x,y)—Volume of x immersed in y
fl(x,y)—x floats in y
sub(x,y)—x sinks in y
in(x,y)—x is in y
Productions: One body, one liquid
P1. in(x,y), Den(x) ≤ Den(y) → fl(x,y) and Vim(x,y) = [Den(x)/Den(y)]Vol(x).
P2. in(x,y), fl(x,y), Wt(x) = w, → Fb(x) = −Wt(x) = −w, so that Net(x) = −w + w = 0.
P3. in(x,y), Den(x) > Den(y) → sub(x,y) and Vim(x,y) = Vol(x).
P4. in(x,y), sub(x,y), Vol(x) = v → Fb(x) = −Vol(x) × Den(y), so that Net(x) = Vol(x) × [Den(x) − Den(y)].
Archimedes' rule
in(x,y) → Fb(x,y) = Vim(x,y) × Den(y)
Productions: Two bodies, one or two liquids
PA. Vol(a) = Vol(b), sub(a,x), sub(b,x) → Fb(a) = Fb(b)
PB. Wt(a) = Wt(b), fl(a,y), fl(b,y) → Fb(a) = Fb(b)
PC. fl(a,y), fl(b,y), Wt(a) > Wt(b) → Fb(a) > Fb(b)
PD. sub(a,y), sub(b,y), Vol(a) > Vol(b) → Fb(a) > Fb(b)
PE. sub(a,y), sub(a,z), Den(y) < Den(z) → Fb(a,y) < Fb(a,z)

The productions shown above are sufficient to solve most simple buoyancy problems.

Analysis of protocols. We will examine verbal protocols of six Chinese students (three experimental and three control) in a Chinese middle school and an American college student at Carnegie Mellon University (who had not previously studied buoyancy). These protocols reveal how the experimental students progressed from almost no knowledge of buoyancy to an ability to solve problems with skill and understanding. They learned to recognize the conditions of particular productions and to elaborate these cues to produce more efficient productions. We first examine some before-instruction and after-instruction tests of the middle school students and then some protocols of the college student.

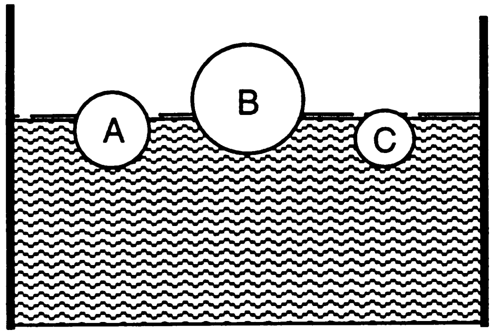
Acquiring basic knowledge of buoyancy. Protocols of the experimental subjects are labeled ES, the control subjects are labeled CS, and the university level subject is labeled US. The study materials and our interpolated comments are printed in boldface type, with the latter in brackets. Key terms in the protocols are italicized. # indicates pause.

Pretest: Experimental and control subjects. Generally, the students of both groups were unable to solve the problems in the pretest. They knew neither the relevant productions nor the cues for selecting them. Instead, they often drew upon "common sense" ideas.

> **Problem 6. [Question] Three balls having the same weight but different volumes all float in a salt solution. Call the buoyant forces exerted on the balls Fb(a), Fb(b), and Fb(c), respectively. Compare the three buoyancies. Why are they related in this way?**
>
> **[The relevant production is PB, Wt(a) = Wt(b), fl(a,y), fl(b,y) → Fb(a) = Fb(b) = Wt(a)].**

As an example, while solving Problem 6 on the pretest, subject ES3 said

> Okay, the *buoyant force* on A is a little *lower* than B and C is lower than A.
> I guess that's A from B,
> A has less, less buoyancy #.
> Buoyancy of A is less than—the buoyancy of B,



Problem 6

it is higher and C is a lot lower in the water. Why? Because they're different heights in the water.
It's, well, I guess B also has a greater *surface area* too. Because they are different, that would probably have an effect on it too.

ES3 estimated buoyancy from a body's perceived height in the water, and sometimes its surface area, equating "buoyancy" with "being buoyed up"—i.e., "floating higher." Another subject, ES1, asserted this explicitly. Other students used the *weight* of a body as the determinant of buoyant force, even when the body was *submerged.* The subjects paid attention to irrelevant conditions and ignored relevant ones.

Posttest. In the posttest, students in the experimental group solved most of the problems correctly and rapidly. They had built appropriate production rules and could recognize relevant cues in the problems. Consider the protocol of ES3 in posttest Problem 2 (boldface comments in the protocols are the authors'; italics mark important terms).

2. [Questions] Mark the following statements True or False:
2.1 There is less buoyant force on a body submerged in shallow water and more buoyant force on a body submerged in deep water.
2.2 There is less buoyant force on a body submerged in high density liquid and greater buoyant force on a body submerged in low density liquid.
2.3 The buoyant force on a hollow iron ball submerged in water is greater than that on a solid iron ball having the same volume.

2.1 a. [Reading problem]
b. That is, "less buoyancy." No, that's false.
c. They're the same. It's the same.
d. *It* **[depth]** *doesn't matter*, because the amount of water is still going to . . . do the same, *displace the same amount.* **[PA: Vol(*a*) = Vol(*b*), sub(*a*,*y*), sub(*b*,*y*) → Fb(*a*) = Fb(*b*)].**

2.2 a. [Reading problem]
b. Okay, it's a *greater density* liquid.
c. And it's going to *submerge and displace a certain amount of water.*
d. But it's going to *weigh more* than the low density.
e. So that's less buoyant, no it's *more buoyant.* That's false.
f. It's going to be more buoyant. **[PE: sub(*a*,*y*), sub(*a*,*z*), Den(*y*) < Den(*z*) → Fb(*a*,*y*) < Fb(*a*,*z*)].**

2.3 a. [Reading problem]
b. "Submerged in," . . ., *submerged*, is going to displace the *same amount.*
c. And it's water; so that's false too.
d. Because it's going to be the same. **[PA].**

For part 2.1, in statement c, ES3 gave the correct answer. Then, in d he gave the reason: He recognized the condition "submerged" (see 2.2b and 2.3b of protocol) and the implied cue "volume of the same body" (2.1c and 2.3b) and thus evoked production PA.

In part 2.2, ES3 found the cues, "submerge" and "density of liquid," and thus evoked PE. Moreover, in c, he inferred the reason from Archimedes' rule, showing that he applied the production with understanding. The same can be seen in part 2.3, where he properly applies PA.

The experimental subjects both recognized the key cues quickly and discarded irrelevant variables. For example, in d in segment 2.1, ES3 notes that "depth doesn't matter."

To illustrate the posttraining differences between the experimental and control groups, we reproduce their protocols in re-solving Problem 6 in the posttest. The relevant production is PB.

The experimental subjects:

ES1: 1. Buoyant force exerted on each ball is the same.
2. **["Why?"]**
3. Because their *weights* are the same. **[PB].**

ES2: 1. All of the three balls have the same buoyancy.
2. Because they have the same *weights.* **[PB].**

ES3: 1. Fb(*a*) = Fb(*b*) = Fb(*c*)
2. Because their *weights* are the same.
3. All of them are *floating* on water. **[PB].**
4. *The buoyancy exerted on each ball is equal to the weight of each one.*
5. They have the same weights.
6. So the buoyancy is the same too.

The control subjects:

CS1: 1. The buoyancies exerted on each ball cannot be compared with each other.
2. Although *weights* of the balls are equal, their *volumes* are unknown. **[Attempts to apply PA].**
3. We cannot get the magnitude of water displaced by each ball, for their volumes are unknown.
4. So we cannot get the buoyancy of each ball.

CS2: 1. Then buoyant forces are equal.
2. Because they have the same *weights.*
3. Based on the formula Vol(*x*) = Wt(*x*), their *volumes* are equal. **[Attempts to apply PA].**
4. Their volumes are equal. According to Archimedes' law, balls with the same volume displace the same volume of water.
5. So the three balls have the same buoyancy.

CS3: Omitted this problem.

All three experimental subjects solved Problem 6 correctly, identifying the key cues, "floating" and "same weight" and ignoring irrelevant variables, such as volume and depth. Then they evoked production PB. In contrast, CS1 erroneously applying PA, thought that he could not find the buoyant forces as the volumes of the balls are unknown. CS2 made a similar wrong inference, although his answer happened to be right. Students in the control group have neither the relevant productions nor the ability to recognize the relevant conditions.

In addition, ES3 provided details of his inference processes in statements 3–5, showing that he understood that, for floating equilibrium, the buoyant force equals the weight of the body.

Elaboration of conditions. As learning continues, subjects distinguish special cases, gradually elaborating the conditions of the productions. Consider US5's protocol on Example D, and Exercises D1–D3. Quotes indicate that the subject is reading the problem statement; blanks filled in by the subject are underlined; # marks pauses; italics mark important terms; brackets indicate conditions and actions of productions, inferences, applications, and interpretations.

Example D:

1. **If a *wooden block floats* on the surface of water, then the *gravity* and the *buoyant force* on the block are in equilibrium.**
2. "Gravity" ##.
3. "Wooden block *floats* on the surface of the water." **[FINDS CONDITIONS]**
4. "Gravity and the buoyant force" #. **[FINDS ACTIONS]**
5. "*Float* on the" ##. **[FINDS CONDITIONS]**
6. Okay. "This means that the buoyant force on the block is equal to the weight of the block." **[P2: fl(block,water) $\rightarrow$ Fb(block) = −Wt(block)].**
7. Yes, okay, that would keep it up above the water. All right. **[EXPLAINS]**

Exercise D1:

1. **If a piece of *ice floats* on the surface of *water*, then *gravity* and the *buoyant force* on the ice are in equilibrium.**
   "This means that the buoyant force on ice is equal to ___________ . . ." **[READS]**
2. "A piece of ice floats on the surface and gravity and buoyant force"—"means that the buoyant force on the ice is equal to" *its weight.* **[ANALOGIZES]** **[P2: fl(ice,water) $\rightarrow$ Fb(ice) = −Wt(ice)].**
3. I guess since the gravity—weight—of the ice. # **[EXPLAINS]**

Exercise D2:

1. **If a *boat floats* in *water* then *gravity* and the *buoyant force* on the boat are in equilibrium.**
2. **This means that the buoyant force on the boat is equal to"** *the weight of the boat.* **[ANALOGIZES]** **[P2: fl(boat,water) $\rightarrow$ Fb(boat) = −Wt(boat)].**

Exercise D3:

1. **Therefore, the buoyant force on the body floating on a liquid surface is equal to** *the weight of the body.* **[GENERALIZES]**
2. **The buoyant force on it may be known so long as its weight is known. [P2: fl(*x*,*y*) $\rightarrow$ Fb(*x*) = −Wt(*x*)].**
3. *Yes*, because it would be equal.
4. The weight of the body. **[EXPLAINS]**

In Example D1, US5 noticed the key cues *gravity* and *floating* and then connected them with the goal. In Exercise D1, she used the same cues, replacing "wooden block" by "ice." Moreover, she translated "gravity" into "*weight.*" She solved D2 faster and more directly. In D3 (2) she generalized the rule to all objects that float, production P2. By analogy, she generalized from specific bodies to objects in general and connected actions with conditions.

We see something similar in another protocol segment involving an object submerged (instead of floating) in two different liquids:

Exercise H1:

1. **A block of iron is placed first in water and then in alcohol.**
2. **The volume of water displaced by the block** *equals* **that of alcohol displaced.**
3. They have alcohol "the weight of water" *blank* "that of alcohol of the same volume." **[READS]**
4. I don't know *which weighs more.* **[SETS SUBGOAL]**
5. "The weight of water," I'd say is *greater* than that of alcohol of the same volume. **[SUPPLIES INFO]**
6. "So the buoyant force on the iron block in the water is" *greater.* **[APPLIES PE]**
7. Weight of water, block of water is greater than in alcohol. **[SUPPLIES INFO]**
8. It's greater than it is. **[PE. sub(*a*,*y*), sub(*a*,*z*), Den(*y*) < Den(*z*) $\rightarrow$ Fb(*a*,*y*) < Fb(*a*,*z*)].**
9. Okay. Water is more dense. **[EXPLAINS]**

Here, US5 used previous knowledge to solve a problem involving different liquids. Needing to compare the weights of displaced liquids of the same volume, she focused on the densities (W = D × V). US5 knows that iron is denser than water or alcohol; hence, it is submerged and the controlling factor is the density of the liquid. As water is denser than alcohol, the corresponding buoyancy is greater. The new production rule is equivalent to PE.

Forming Subgoals. When Ss were asked to explain their solutions and solve related subproblems, they elaborated the conditions of their productions, frequently adding goals. We first examine US5's Exercise F1 and F2.

Exercise F1:

"Are same or are different?" "If an *iron* block and *copper* block having the *same volume* are *submerged* in water, the magnitudes of the *buoyant force* on them" **[READS]**
—iron block and copper block, same *volume.* **[FINDS CONDITION]** They'd be *the same* **[APPLIES PA]**
"because Archimedes' principle" says so. **[EXPLAINS]**
Buoyancy liquids displaces, the same liquid (inaudible). Okay are the *same.* **[CHECKS]**

Exercise F2:

"Since the *iron* and *copper* blocks of *equal volume* are *submerged* in water," # "*blank*" of water is displaced and in both cases the **[READS]**
displaced water has" *the same weight.* **[APPLIES PA]**
"According to" (inaudible) # # "Archimedes' rule, the magnitude of buoyancy on a body is equal to"—"the buoyancy of the metal blocks," *okay.* **[READS]**
"Since the iron and copper blocks submerged in water"—water "of water is displaced"—the ______ amount of water—*same* "amount of water is displaced." And in both cases displaced water has the same weight. "According to Archimedes' rule, the magnitude of buoyancy on a body is equal to" the *amount of water displaced* "so **[READS]**
the buoyancies on the metal blocks" are *the same*, are the same magnitude. Same volume, water **[INTERPRETS]**
(inaudible) are equal.

In Exercise F1, US5 goes from the volumes of bodies to the volumes of water displaced and then to the buoyant forces. She then notes that the solution amounts to applying Archimedes' rule. We hypothesize that in Exercise F2 she followed the three stages of the problem statement, which took her from the goal of comparing buoyant forces to the goal of comparing weights of water displaced, then to the goal of comparing volumes of the bodies immersed.

PG: $\text{Goal}[\text{Fb}(x,1) - \text{Fb}(y,1)],\ \text{sub}(x),\ \text{sub}(y) \rightarrow \text{Subgoal}[\text{Wtd}(1,x) - \text{Wtd}(1,y)]$

PH: $\text{Goal}[\text{Wtd}(1,x) - \text{Wtd}(1,y)] \rightarrow \text{Subgoal}[\text{Vim}(x,1) - \text{Vim}(y,1)]$

PI: $\text{Goal}[\text{Vim}(x,1) - \text{Vim}(y,1)],\ \text{sub}(x),\ \text{sub}(y) \rightarrow \text{Subgoal}(\text{Vol}(x,1) - \text{Vol}(y,1))$

Here we have a first example in her protocols of the incorporation of goals among the conditions of productions in order to guide search.

In subsequent exercises in part F, the subjects learn to disregard irrelevant variables like depth and weight when objects are submerged and to determine volume when weight and density are given.

Forward and Backward Solution. Let us return once more to Problem F:

> **IF two bodies with unequal weight but the same volume are submerged in the same kind of liquid, THEN are the buoyant forces on them equal?**

An (expert) solution, ignoring the irrelevant weight, is summarized in a production, related to PA.

$$\text{sub}(a,x),\ \text{sub}(b,x),\ \text{V}(a) = \text{V}(b) \rightarrow \text{Fb}(a) = \text{Fb}(b).$$

For the subject who has just learned Archimedes' rule, solving this problem would involve at least seven steps:

First, attention must be drawn by the goal to the buoyancies. Buoyancies draw attention to weights of displaced liquids; weights draw attention to volumes and densities. Densities call attention to the statement that the liquids are the same; volume calls attention to the statements that the objects are submerged and have the same volumes. Finally, the information about the volumes of the objects and the identity of the liquids implies that the volumes and weights of the displaced liquids are equal—hence, also the buoyancies.

Unless the successive shifts in attention take place at the proper times, the conclusion will not be reached without search. These shifts in attention can be activated by learnable goal-driven productions; they do not occur to naive learners, however good their basic perceptual abilities. For the more expert problem solver, recognizing that the objects are submerged, that the liquids are the same, and that the objects have the same volumes suffices to evoke from memory the required production and produce an immediate solution. The novice's specific productions have been *chunked* by the expert into a single production that goes directly from the relevant conditions to the conclusion.

Problems can be solved by working forward from the given quantities or working backward from the goal (3). For example, a slightly modified form of production PA, working forward, can compute the buoyant force on y, given that x and y are known to be submerged and have the same volume, and that the buoyant force on x is known:

$$\text{PA*: Vol}(x) = \text{Vol}(y),\ \text{sub}(x),\ \text{sub}(y),\ \text{Fb}(x) = K \rightarrow \text{Fb}(y) = K.$$

If no single production exists whose action solves the problem, and whose conditions all correspond to known facts, working forward will generally proliferate search with no clear rudder to steer the process. Nevertheless, for easy problems, experts frequently work forward without setting goals, confident that an answer will be found quickly. In more difficult problems experts fall back on working-backward methods, using goals to guide their path. For example, by modifying PA* further, we can incorporate it in a working-backward scheme, which will be evoked only if the goal is already set

$$\text{PA\#: Goal[Find Fb}(y)\text{]}, \text{Vol}(x) = \text{Vol}(y),\ \text{sub}(x),\ \text{sub}(y),\ \text{Fb}(x) = K, \rightarrow \text{Fb}(y) = K.$$

When a goal is present but essential facts are not available for satisfying the conditions of the production, then a subgoal must be created to find one or more of these facts.

Composition. After Ss had learned how to incorporate goal structures in productions, as in Exercise F1, above, they then were helped toward composing the new productions into longer sequences, suppressing intermediate steps. To this end, they were given additional true–false questions, as in Exercise F3, answerable directly by applying the new productions. Here is part of the protocol of US5 on Exercise F3:

Exercise F3: If a steel plate and a hollow steel ball, both having the same volume, are submerged simultaneously in the same kind of liquid, then: (Problem 1) the buoyancy on the steel plate is greater.

That's false cause they have the same volume. They displace the same amount of water. **[PA].**

In Exercise F3.1, US5 mentions the volumes of bodies and of water displaced, suggesting that, starting with the goal of comparing the buoyancies, she was led from PG through PH to PI. These three productions can be composed into

$$\text{PK: Goal[Fb}(x,1) - \text{Fb}(y,1)\text{]},\ \text{sub}(x),\ \text{sub}(y) \rightarrow \text{Subgoal[Vol}(x) - \text{Vol}(y)\text{]}.$$

In Exercise F4, US5 failed to infer equality of volume of two bodies from equality of weight and material. Apparently, she did not then have a production available that would have permitted her to make this inference.

US5's protocol in Exercise F6.1 shows that she is now able to disregard irrelevant variables like shape and depth.

Exercise 6: If a lead block, an iron ball, and an aluminum block, all having the same volume but different shapes, are submerged in water to different depths, then: (Problem 1) the buoyant force on the aluminum block is the greatest.

That's false because they're all the same volume and the depth doesn't matter.

Tuning. Generalizing by composing productions can speed up problem solving but may also overspecify the condition side of the new production. By working through additional problems, Ss generalized the productions to a proper level. While doing problem sequence G, Ss turned PI,

$$\text{PI: Goal[Vim}(x,1) - \text{Vim}(y,1)\text{]},\ \text{sub}(x),\ \text{sub}(y),\ \text{Vim}(x) = \text{v1},\ \text{Vim}(y) = \text{v2} \rightarrow \text{Subgoal(v1} - \text{v2)}$$

into a new, more general, production,

$$\text{PL: Goal[Fb}(x,1) - \text{Fb}(y,1)\text{]} \rightarrow \text{Subgoal[Vim}(x) - \text{Vim}(y)\text{]}.$$

PL deletes from the condition side of PI "submerged" and sets as subgoal "volumes of their submerged parts." This change greatly expands its application range at the expense of generalizing the goal structure. The usefulness of the generalization depends on the distance of the variables in PL from the problem goals and givens.

After the Ss had built fundamental productions about buoyancy, they developed further their abilities to recognize and use special conditions. We look at US5's Exercise G13, which concerns an iron and an aluminum ball of the *same weight*, both *submerged* with *buoyant forces* Fb(Fe) and Fb(Al), respectively:

Exercise G13:

1. [Reading problem].
2. Lets see, "the *same weight.*" "*Simultaneously submerged.*" **[PK]**
3. Does it mean it's the *same size*? **[FINDS CONDITIONS]**
4. Let's see. Aluminum weighs less than iron **[PI]** [i.e., is less dense]—**[SUPPLIES INFORMATION]** that would be greater, *greater volume.* **[V = W/D]**
5. "Simultaneously submerged in kerosene" then Fb(Fe), which is iron, is less than Fb(Al)?
6. Let's see. Fb(Fe) iron, caused the aluminum

displaced a greater amount. Okay, Fb(Fe) is less than Fb(A).

Here, PK was triggered when US5 detected "submerged." Then PI was evoked according to the subgoal of PK, and this solved the problem.

For another example, consider Exercise G16, in which a *wooden* and an *ice* block of the *same weight* are *floating* on water:

Exercise G16:
1. Reading problem.
2. "*Wooden* block of the *same weight*."
3. Think the *ice is more* **[dense]** *than wood.* This ## last time.
4. The *buoyant force*, float higher.
5. "The buoyant force" would be less than that of ice. "The *volume* of water displaced by wood" is less than "by ice." Okay less.
6. (Reads the answer) "Is equal to." "*Same weight*!" "Wooden block and ice of the same weight are *floating* on," "*floating* on the surface."
7. "Then the buoyant force on "..." and that of ice" are the *same*, okay. All right, I guess it's the same.

In this problem, US5 did not note initially the cue of "floating" (statement 1). Perhaps influenced by the apparently similar problem G5 (statement 3), she tried to apply PI first (statements 4 and 5). Unfortunately, the conditions of PI are not satisfied here. Upon feedback of the error, she reread the problem (statement 6), noted the condition "floating," and solved the problem with PB.

Conclusion

We have shown, using examples from protocols, how students acquire productions for solving buoyancy problems. The laws of buoyancy appear mainly on the right-hand (action) sides of the productions, while the conditions for applicability appear on the left-hand sides. Most of the students' time is devoted to acquiring the condition sides and learning to notice cues that signal when conditions are satisfied.

A very small set of productions suffices for solving all the standard buoyancy problems, but students accumulate a much larger set. Initially, they learn the relevant conditions in simple situations and learn to ignore irrelevancies. As they begin to deal with problems requiring several steps of search, they begin also to acquire productions that control search by including goals and subgoals among their conditions and actions.

Finally, students gradually compose and "tune" their productions to solve common problems in a few steps each, or even a single step. If some productions that students assemble are forgotten over time, the redundancy permits them to be recreated from those that remain.

The behavior of students solving buoyancy problems, and their focus on the acquisition of the condition sides of productions, is not peculiar to that domain. The same behavior has been observed in other domains when students learn from worked-out examples and problems.

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1. Anderson, J. R. (1983) *The Architecture of Cognition* (Harvard Univ. Press, Cambridge, MA).
2. Chi, M. T. H., Bassok, M., Lewis, M., Reimann, P. & Glaser, R. (1989) *Cognit. Sci.* **13,** 145–182.
3. Simon, H. A. & Simon, D. P. (1978) in *Children's Thinking: What Develops?*, ed. Siegler, R. (Erlbaum, Hillsdale, NJ), pp. 325–347.
4. Zhu, X. (1983) *Acta Psychol. Sinica* **1,** 9–18.
5. Zhu, X. & Simon, H. A. (1987) *Cognit. Instruct.* **4,** 137–166.
6. Neves, D. M. (1978) in *Proceedings of the Second Conference on Computational Studies of Intelligence* (Can. Soc. Computat. Studies Intelligence, Toronto), pp. 191–195.
7. Anzai, Y. & Simon, H. A. (1979) *Psychol. Rev.* **86,** 124–140.
8. Anderson, J. R., Boyle, C. F., Farrell, R. & Reiser, B. J. (1987) in *Modelling Cognition*, ed. Morris, P. (Wiley, New York), pp. 93–133.
9. Anderson, J. R., Corbett, A. T., Koedinger, K. R. & Pelletier, R. (1995) *J. Learning Sci.* **4,** 167–207.