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ROLE OF POSITIONING

The role of positioning in constructing an identity in a third grade mathematics classroom[i]

INTRODUCTION

> Whatever we offer in the classroom becomes an opportunity to pursue this longer-term agenda of identity building; our primary affective engagement is with this agenda, with becoming who we want to be, not with learning this or that bit of curriculum. (Lemke, 2000, p. 286)

We often encounter the term "identity" in studies on learning. Why has the concept of identity become so popular in educational research? We may find an answer to this question in Lave and Wenger's (1991) influential book, Situated learning: Legitimate peripheral participation. This book significantly influenced ways of conceptualizing learning. Lave and Wenger described learning as the transformation from a newcomer to an old-timer in a particular community through participating in that community's practices. Examples taken from ethnographies of traditional apprenticeships and more recent affiliative groups, such as Alcoholics Anonymous, were used to illustrate this view of learning as participation. Lave and Wenger argued that learning, or becoming an active participant in a community, is strongly connected to the identity-building of learners. For example, attendees of meetings of Alcoholics Anonymous demonstrate their new affiliation with this group by referring to themselves as "recovering alcoholics" and rejecting their previous identity as "social drinkers."

Until recently, many educators have viewed learning as the acquisition of knowledge. As Gergen (1999) has noted, since North American public education is grounded in Enlightenment thought, its aim has been to educate individuals to make rational decisions based on factual information. In order to pursue this goal, teachers have encouraged students to memorize facts and procedures and apply that information when they solve academic problems. Thus, learning is viewed as an individual activity, and the information acquired becomes an individual's possession (Sfard, 1999).

The participation metaphor defines learning very differently. People must learn the norms, values, and practices of each new community they enter. If one sees a classroom as a community of learners, students can demonstrate their learning through active participation in the classroom's social and intellectual practices. Learning, in this case, is a more collective than individual activity (Sfard, 1999). In addition, this version of learning implies that identity formation is an integral aspect of engagement in classroom practices. By conforming to the classroom

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norms and practices, a student shows that he or she is a legitimate member. In this way, his or her identity as a “competent,” “advanced,” or “struggling” student may be formed.

The aim of this chapter is to explore the relationship between students’ learning and their identity-building in a North American, third grade mathematics classroom, using notions from discursive psychology (e.g., positioning). First, we will explain the concept of positioning, and then discuss why positioning is appropriate for investigating identity. We will use ethnographic data we collected to illustrate how positioning could be used as a tool for interpreting students’ identity-building as learners of mathematics. Finally we will discuss the implications of our work for educational research on identity.

POSITIONING: WHAT IS IT?

Positioning and its roots

In recent years, psychology has taken a discursive turn in both theory and method: the “second cognitive revolution” according to Harré and Gillett (1994, p. 18). What is most significant about this theoretical shift is the orientation to discourse and context. Positioning theory aims to explain the relationship between discourse and psychological phenomena. Harré and van Langenhove (1999) viewed positioning as a more dynamic form of social role. Participants in conversations take on certain roles, such as speaker, active or passive listener, opponent of the issue being discussed, and so on. However, we should keep in mind that conversations are on-going discursive practices in which storylines and participants’ roles are subject to change as conversations evolve. The participants may not keep the same role from the beginning to the end, but they assume different kinds of roles during the conversation. Considering changes in participants’ roles, it seems quite relevant to use “positioning” in order to describe the dynamics of discursive practice.

Positioning is defined as “the discursive process whereby people are located in conversations as observably and subjectively coherent participants in jointly produced storylines” (Davies & Harré, 1999, p. 37). One can be positioned by another or by oneself, interactive or reflective positioning, respectively. This definition means that participants position themselves or are positioned in different conversational locations according to changes in storylines. A tri-polar relationship between position, storyline, and speech act is essential for conversation, and is the conceptual base of positioning theory (van Langenhove & Harré, 1999).

Positioning theory has begun to provide a useful framework for analyses of classroom discourse and its dynamics. For example, Ritchie (2002) used positioning to investigate the dynamics of students’ interactions within same-gender and mixed-gender groups during science activities. Ritchie concluded that one student, for instance, may struggle with multiple positional identities (i.e., boss, good student, and victim), which could be displayed in different social contexts. Positioning analysis revealed the complicated nature of interpersonal relationships during group work in a classroom.

The theoretical concept of positioning can be assessed in a variety of ways. For example, pronoun use in conversations can indicate how a person aligns others and/or the self. By using “we” instead of “you,” a speaker positions herself as part of a group that includes the listener. If the pronoun, “he” is used instead of “you” and the referent person is present, then the speaker may be informing an audience about the listener as in, “he seems upset.” In this instance, the speaker and the audience are part of a group that is observing and commenting on the listener. Another way to evaluate positioning is through revoicing. That means of assessing positioning will be discussed next.

Revoicing

The word revoicing first appeared in classroom studies by O’Connor and Michaels (1993, 1996). According to their definition, revoicing refers to a “particular kind of re-uttering (oral or written) of a student’s contribution ----- by another participant in the discussion” (1996, p. 71). O’Connor and Michaels cited several functions of revoicing. First, revoicing can be used to reformulate a student’s utterance: a teacher may slightly modify what a student has said for the purpose of confirmation or clarification. Second, revoicing can be employed to create alignments and oppositions during an argument[ii]. Some varieties of revoicing involve a “warranted inference,” which was originally used by Schiffrin (1987, cited in O’Connor & Michaels, 1993, 1996). A warranted inference can usually be recognized by linguistic markers such as “so.” By using “so,” a speaker can associate his or her utterance with what was previously said by another participant in a conversation. For example, the following conversation taken from O’Connor and Michaels (1993, p. 322) between Steven (student) and Lynne (teacher) describes how with the use of “so” revoicing created alignments and oppositions during an argument:

> Steven: … um, but if she kept her, um, sugar and used that, and then took her things of ten to twenty-two and just picked another number like halfway like Allison said and then just made that her concentrate.
>
> Lynne: So then, you don’t agree with Sarita that if she picks a number halfway between that that’s not really making her first concentrate.

Another example of warranted inference from O’Connor and Michaels (1993, p. 323) is as follows:

> Zelda (student): They live in the Northeast.
>
> Debby (teacher): Oh, okay. So you have a lot of family up in the Northeast.

In the first excerpt, Lynne used “so” to align Steven’s and Sarita’s explanations. Although Steven did not mention Sarita in his utterance, Lynne’s revoicing succeeded in locating Steven in an oppositional position to Sarita. That is, each of them was given a different stance by the teacher in terms of their problem solutions. In the second example, the word "so" was effectively used by Debby (teacher) to recast the previous utterance of Zelda (student), which may not have been clear to her classmates.

In both of these instances, the pattern of teacher-student interaction is quite different from the ubiquitous pattern of whole class recitations, the Initiation-Response-Evaluation/Feedback (I-R-E/F) (Mehan, 1979; Wells, 1993). In the I-R-E/F sequence, the teacher is the center of power and authority. It is a teacher who controls the topic of conversation, the allocation of conversational turns, and the evaluation of answers to questions. Students have almost no agency in this kind of conversation because their talk is restricted to responding to the teacher's questions. Both of the above examples, however, show that the students (e.g., Steven and Zelda) have more agency when revoicing occurs. O'Connor and Michaels (1993) argued that revoicing gives students more room to accept or reject the warranted inference by the teacher.

There is one more important aspect of revoicing. As we can tell from the examples, revoicing functions as a means of positioning. In the first example, for example, Lynne (the teacher) used revoicing and restated Sarita's explanation to draw a comparison between her explanation and that provided by Steven. In this way, Lynne may be helping her students identify the mathematical argument as well as clarify two distinct perspectives on that argument (cf. Forman, Larremendy, Stein & Brown, 1998).

Why is positioning a useful methodological tool?

If learning is conceptualized as participation, then what is learned are the norms and practices of a community (Sfard, 1999). In schools, students and teachers participate in collective work to construct, maintain, or alter the cultural and historical practices of their classroom community. To view learning as participation requires us to recognize the importance of members' social relationships in the community, which are also critical to their identity formation (Lave & Wenger, 1991). In addition, when we begin to examine the dynamics of participation in classrooms, we need to keep the teacher's long-term and short-term instructional goals in mind as well as the students' goals. In the examples provided in the book by Lave and Wenger (e.g., apprentice butchers) we see that many individuals may hold a long-term goal of becoming an expert in their field. Nevertheless, to apply the participation metaphor to a classroom setting, a framework like positioning is needed to understand interactions at the local (or micro) level (Linehan & McCarthy, 2000).

Tri-Polar Structure of Conversations

Positioning theorists believe that a conversation has three constituent and interactive elements: position; the social force of the speech-act; and storyline (van

Langenhove & Harré, 1999). This tri-polar structure enables us to see that one takes up a certain position during a conversation, and that positioning is necessary to make conversations possible. Through positioning, participants are given a role and moral order, which are embedded in the particular position they take up. In a classroom, students can be various kinds of actors in discourse: listeners, contributors, supporters, facilitators, manipulators, opponents, and so on. Identifying such positions can help us to see not only who is playing what role(s) but also how participants relate to each other. This also enables us to understand the dynamics or power relations of the classroom community.

Viewing a conversation according to this tri-polar grid also helps to identify community norms especially through storyline(s) appearing in a conversation. Linehan and McCarthy's (2000) study, for example, revealed that respecting the teacher's authority generated a storyline as a classroom norm, and that one female student resisted the teacher by resisting the storyline already established and creating her own storyline. Linehan and McCarthy's framework made the tensions and/or dilemmas occurring during classroom conversations intelligible.

Positioning as a fundamental aspect of identity

Positioning theory contributes to our understanding of identity, which is essential to learning in the participation framework. Two notable definitions of identity are: "to be recognized as a certain kind of person by others" (Gee, 2001, p. 99) and a "collection of stories about persons, or more specifically, those narratives about individuals that are reifying, endorsable, and significant" (Sfard & Prusak, 2005, p. 16). Unlike an essentialist view of identity (i.e., identity as a fixed, inherent attribute), these definitions have been influenced by discursive psychology (cf. Bucholtz & Hall, 2005; Holland, Lachicotte, Skinner, & Cain, 1998).

For the purpose of exploring identity, positioning theory may provide one of the overarching analytic means for understanding how social and psychological phenomena manifest themselves in discourse. It views a conversation as interaction of position, speech-act, and storyline, and this conceptualization enables us to see conversations in terms of participant roles and alignments. A closer look at such roles and alignments will enable us to clarify identities that appear or are constructed through discourse. Revoicing often works as a means of positioning. Appreciating these potentials of positioning theory, we will illustrate them in the next section with examples from a study of students' identity-building in a third grade North American classroom.

METHODOLOGY

Settings and Participants

Ethnographic data were collected in a private elementary school located in an urban neighbourhood in a north eastern state in the United States. Most of the students were from upper-middle class backgrounds and their parents were highly educated. The class consisted of 17 students (between the ages of 8 and 9 years), 7 girls and 10 boys. Mrs. Porter[iii], the teacher, had 30 years of teaching experience. Ten students (3 girls and 7 boys) were European-American, 4 children (1 girl and 3 boys) were African-American, and 3 students (1 girl and 2 boys) were Asian-American. The students chose where to sit at five tables: the resulting table groups were same-gender. These groups were stable during most class lessons, although, occasionally, students were observed sitting at other tables or on the floor to work alone or with a partner.

In this analysis, we focused on two target students: Ophrah and Pulak. These two students were selected, in part, because their behaviors were frequently recorded in our dataset. This wealth of data provided us with the possibility of confirming or disconfirming hypotheses from one case to the other, consistent with a replication logic that is appropriate for multiple case studies (Yin, 2003). As Yin argues, this strategy has the potential for producing more robust findings than can be achieved using a single case.

Ophrah was a tall, mature-looking, well-dressed and out-going African-American girl. She frequently volunteered to share her solutions during whole class discussions. When she was working in a small group, Ophrah usually assumed a leadership position. Mrs. Porter's ratings of Ophrah's mathematics proficiency changed from average (in the fall) to above average (in the winter). Pulak was a short, thin, introverted, and serious-looking boy of South Asian descent. Pulak's older brother had been a student in Mrs. Porter's class the previous year. Mrs. Porter rated him as a highly proficient mathematics student in the fall and winter. In addition, when the students were asked to describe, in writing, a "good math thinker" in midwinter, three of them mentioned Pulak (no other student was named). Thus, he was recognized by his peers and the teacher for his proficiency in mathematics.

Mrs. Porter's Profile and Teaching Philosophy

Mrs. Porter had taught at this private school for more than 20 years. She did not follow a formal mathematics curriculum but used a range of instructional materials and her personal teaching journal as a guide. She used a developmental approach to enhancing students' learning by encouraging them to use problem-solving strategies that made sense to them. In order to accomplish this, she encouraged the students to articulate their thinking processes and justify their results to themselves and others. Since she focused on children's learning processes, Mrs. Porter often told the students not to erase what they wrote when they worked on a task. She also made it clear that she was interested in their problem-solving strategies, not just their answers. She frequently asked her students to explain and compare their strategies with their classmates during whole class presentations.

In the winter, Mrs. Porter wrote a newsletter to her students' parents to explain her teaching philosophy. It was entitled a "Reform Mathematics Teaching Philosophy: What I Believe about Teaching Mathematics." In the newsletter, she emphasized the importance of sense-making for mathematical concepts, rules, and procedures. She proposed that students should be encouraged to think and develop problem-solving strategies that are meaningful. She clearly stated that knowledge could not simply be given to students but should be constructed by students for themselves. She stressed that communication was a highly valued element in a successful math program. Thus, Mrs. Porter aligned herself with a reform storyline that valued active student engagement in meaningful problem solving.

Data Collection

Ethnographic data were collected by the second author (and two graduate student assistants) during the students' math instruction (twice a week) for four months in the beginning of the school year (September-December). The classes were observed and audio recorded. The field notes were used to help create transcripts from the audiotaped lessons. Interviews with the teacher were also conducted, audio recorded, and transcribed. Classroom artifacts such as student surveys and mathematical problem solving work were also available. We asked the teacher, early in the fall and winter, to rate each student in her class on their mathematical proficiency.

Overview of Data Analysis

Our approach to data analysis was based on an ethnographic logic of inquiry (Green, Dixon, & Zaharlick, 2003). This is not a linear approach to data analysis. It goes through a series of cycles or phases to pose questions, represent data, and analyze events. In addition, contrastive analysis is required as a way to validate interpretations through triangulating perspectives. Using different types of artifacts (field notes, transcripts, student work, interviews) is one way to triangulate. Another way is via member-checking (asking a participant in the study if the researchers' interpretation "rings true") (Toma, 2006, p. 413). In the past, we shared our completed analyses with the classroom teacher and included her responses in our publication (Forman & Ansell, 2001). Finally, a holistic perspective is required, which means demonstrating how the parts of the analysis relate to the whole. One tool for displaying the parts and the whole is the transcription (Ochs, 1979).

In the analysis reported in this chapter, we compared and contrasted the interactive and reflective positioning of two students (Ophrah and Pulak) during whole class discussions. (See Forman & Ansell, 2001, 2005 for other analyses based on this dataset.) In particular, we identified segments of discourse in which revoicing occurred between the teacher and each of these two students, while the rest of the class participated as a silent or vocal audience. Our hypothesis was that these instances of positioning might help us understand the formation of the students' mathematical identities. We will report our findings in three parts: first, we will summarize the classroom norms; second we will examine how Mrs. Porter positioned Ophrah and Pulak (i.e., interactional positioning); third, we will describe how Ophrah and Pulak positioned her- and himself (i.e., reflective positioning).

RESULTS

The Norms in Mrs. Porter's Mathematics Classroom

Mrs. Porter articulated a vision of mathematical proficiency to her students during the first week of school:

> I want to tell you something about good math students and good mathematicians. They are not people who always get right answers quickly. They are people that have memorized some things and that's what they know. A mathematician makes hundreds of mistakes. Because a mathematician is always trying to find new ways of doing things. . .Was he (Einstein) a bad mathematician? No, he was a good one. And he became one of the most famous scientists and mathematicians in the world, because, not because he always had the right answer. But because he never gave up. . . So, trying is what's most important.

Thus, for Mrs. Porter, accuracy and speed were less important indices of proficiency than diligence, flexibility, and courage. This statement helped us refine our notions of the teacher's instructional goals and the mathematical proficiency storyline. If students were going to conform to her reform mathematics goals, then they would need to demonstrate that they would be willing to work hard, persist, and learn from their mistakes.

Interactional / Reflective Positioning of Ophrah and Pulak

We chose two lessons in September, one in October, and one in December to illustrate our investigation of the relationships between positioning and identity formation for several reasons. First, these four lessons illustrated well the routines of whole class instruction (participation structures) and storylines in this classroom. Second, we wanted to explore whether the patterns that occurred early in this classroom persisted for at least four months. This is important because identity formation takes place over significant periods of time (Lemke, 2000; Wenger, 1999).

On September 10, the students in Mrs. Porter's classroom worked on the following problem: You read for 15 minutes a day. How much time will you have spent reading in one week? Some students chose 5 days for a week, and others chose 7 days for a week. Ophrah, who chose 5 days, was the first student who volunteered to share her strategy.

Excerpt 1: An eager student with useful information

Mrs. Porter	…. Who would like to tell us, not the answer, how they started. Ophrah.

Ophrah	I multiplied 15 times 5.
Mrs. Porter	You multiplied, and do you, how did you do that?
Ophrah	I wrote the15's time's tables on the back of my paper and then I figured it out.
Mrs. Porter	Did you, OK, can I see how that looks?
Ophrah	Right here.
Mrs. Porter	OK. So Ophrah said, she knew she needed 5 fifteen's, Bernard. And this is something some of you might want to write down. 15 times 5. So first you wrote, say it.
Ophrah	On the back, 15 times zero.
Mrs. Porter	15 times zero equals
Ophrah	Zero
Mrs. Porter	Zero
Ophrah	15 times 1 equals
Mrs. Porter	15 times equals
Ophrah	15.
Mrs. Porter	15.
Ophrah	15 times 2 equals 30.
Mrs. Porter	And of course (…….) and two 15's is 30.
Ophrah	15 times 3 equals 45.
Mrs. Porter	How did you know that?
Ophrah	Uhm
Mrs. Porter	Do you know these time's tables? Did you memorize them? Or did you figure it out?
Ophrah	I figure them out.
Mrs. Porter	How did you figure it out?
Ophrah	Uhm, I just thought in my head like, that 1 and 5, that 15, something like that.
Mrs. Porter	OK. And then when you got to 30. Oops, when you got to 30, how, how did you know it, the next one was 45?
Ophrah	Uhm
Mrs. Porter	You said 30, and then what did you say? Try to figure it out.
Ophrah	30 plus 15 is 45.
Mrs. Porter	How did you know that?
Ophrah	I think I just did.
Mrs. Porter	You just did. You didn't say, 15 and 10 more and then 5 more? You just knew that. OK. Then 15 times 4.
Ophrah	Then 15 times 4 equals 60.
Mrs. Porter	Is 60, right?
Ophrah	Uh-huh. Then 15 times 5 equals 75.
Mrs. Porter	Equals 75. And then you got the 5 days. We'll stop there. So, that would be 75 minutes. M-I-N-U-T-E-S. How many people when they got this, and most of you who did, and most of you who did 5, days got 75, wrote the word minutes after it?

The excerpt above shows that Ophrah had first written the multiples of 15 on her paper, instead of using the standard multiplication algorithm (15×5=75). To use this standard algorithm, students have to understand regrouping, which refers to carrying and borrowing in addition and subtraction. In this excerpt, Mrs. Porter revoiced Ophrah almost line by line. Therefore, the sequence was quite long. Since this was the first day of school, it seemed that Mrs. Porter wanted to informally assess Ophrah's understanding of multiplication, an important instructional objective in third grade. The fact that Ophrah was the first student to volunteer showed her willingness to reflectively position herself as an eager student. Mrs. Porter's suggestion to Ophrah's classmates ("And this is something some of you might want to write down") positioned her as providing useful information. Thus, this exchange indicated Ophrah's conformity to the reform storyline in which students actively engage in communicating their solution strategies to other members of the community.

The following excerpt comes from the same lesson in the previous excerpt. Pulak used the standard algorithm and showed that he was capable of regrouping.

Excerpt 2: An advanced student

Mrs. Porter	Before we get to 7 times did anyone else do 5 times another way? Started a different way? And oh yes. OK so you did, tell us.
Pulak	15 times 5.
Mrs. Porter	He did 15 times 5 this way. Miranda. Ophrah said that she did 15 times 5 too. And she did. And to figure it out, she did it this way. When Pulak figured it out, he did it a different way. And ah, Pulak show us what you did.
Pulak	I did 5 times 5.
Mrs. Porter	Uhm-hum.
Pulak	And I
Mrs. Porter	What did you write down?
Pulak	I took a pen.
Mrs. Porter	First, did you put something down here?
Pulak	Oh ya, the 5.
Mrs. Porter	OK.
Pulak	And I put a 2 up there.
Mrs. Porter	Which is really 20. Yes.
Pulak	Then I, uhm, did 1 times 5 is 5 and I did 5 plus 2 is 7.
Mrs. Porter	And you got the same answer. That's the way a lot of your parents would do it. Because that's the only way we were allowed to do it in school. That doesn't mean it's the right way. And it's a very confusing way to a lot of people. Especially for people for whom regrouping is difficult. So, if you want to do it because you understand it and it's a good way for you, great. If not, do it a way that makes sense to you. I figured you'd know that way because your brother does it that way too. I know that from last year.

One of the striking things in the above excerpt was Mrs. Porter's last utterance. She mentioned Pulak's older brother and implied that Pulak had already learned about regrouping from him. Moreover, she used the pronoun "we" ("Because that's the only way we were allowed to do it in school"), which meant people of previous generations. This implies that Pulak was aligned with his older brother and adults in terms of mathematical proficiency. Then, she said that the standard algorithm, which required regrouping, was too difficult for most of her students to understand. As a result, Pulak was interactively positioned by Mrs. Porter as an advanced student because of his knowledge of regrouping.

In addition, Mrs. Porter did not revoice Pulak line by line as she had in the pervious exchange with Ophrah. Thus, she chose not to clarify or broadcast his advanced strategy to his classmates. This different discursive routine (absence of revoicing; referring to adults and older siblings; and to conventional mathematics strategies such as regrouping) may represent a second storyline in this classroom. Perhaps failure to conform to the reform storyline meant that Pulak was positioned as more advanced than the rest of his classmates. In this instance, nonconformity to classroom norms was not negatively sanctioned but may have been (unintentionally) rewarded.

Later in the lesson, the students who assumed that reading 15 pages a day occurs throughout the 7 day week (not just on the 5 school days) presented their solutions. Another student, Raj, presented the standard algorithm he used to solve the problem. Mrs. Porter's interactions with Raj were similar to those with Pulak in that she did not unpack his explanation for the rest of the class and highlighted sense-making in her response at the end of his explanation: "OK. So, you did it exactly the same way he (Pulak) did. You used that algorithm. OK. And you had 105 minutes. That's one way that's possible to do it. Again, if it makes sense, don't try it if it doesn't." Here, Mrs. Porter aligned Raj with Pulak by identifying Raj's strategy as the same as that used by Pulak, and that this approach, only one possible way to solve the problem, should be used only if it makes sense.

Near the end of this segment of the lesson, another student, Nathan, for whom the standard algorithm apparently did not make sense, expressed his desire to understand Pulak's use of the multiplication algorithm. Nathan said, "I don't understand what Pulak did." Mrs. Porter replied, "You're right, you probably don't. And that's OK. I wouldn't write that down if you don't understand it. Because it's a tricky, complicated thing to do. But Pulak understands it. And Raj does. So, it's a really good thing for them to use, 'cause it's quick. But quick isn't always the best."

In this instance, we may be seeing two storylines emerge: a reform storyline in Mrs. Porter's conversations with Ophrah and Nathan, and a conventional storyline in her conversations with Pulak and Raj. If this is the case, then students like Ophrah and Nathan could be viewed as conforming to the reform storyline if they use invented strategies that make sense to them as well as communicate those strategies to their classmates (with the teacher's help). In contrast, Pulak and Raj could be seen as conforming to a different storyline: one that privileges efficiency and speed and aligns them with older people. In this storyline, accuracy is valued more than clarity of communication. Thus, it appears that as early as the first week, two students were positioned by the teacher as advanced in their proficiency because they failed to conform to her articulated reform storyline. If our hypotheses about the two storylines and the two types of participation structures are useful (revoicing vs. no revoicing), then we should be able to test them by looking at later lessons using tasks that could be solved using invented strategies or conventional algorithms.

In another lesson, on September 21, the students worked on the following problem: The first post office in the U.S. was established in 1789. How long ago was that? Since the data were collected in 1998, the expected answer was 209 (1998-1789=209). Ophrah did not use the standard algorithm which required subtraction and regrouping. She used addition to answer the problem, adding tens to 1789 until she reached 1998. Unfortunately, she made an error during this multi-step process.

Excerpt 3: Developing skill

Mrs. Porter	OK. Another strategy? Ophrah?
Ophrah	Well, I started with 1789 plus 10.
Mrs. Porter	1789 plus 10.
Ophrah	Equals 1799.
Mrs. Porter	1799.
Ophrah	And 1799 plus 10 equals 1809.
Mrs. Porter	1809?
Ophrah	Yes.
Mrs. Porter	And you kept adding by tens?
Ophrah	(not audible)
Mrs. Porter	You thought what?
Ophrah	I messed up on it. I messed up on 1799….
Mrs. Porter	Ahh. So you missed one ten. And then when you finished all of your adding, did you count up the tens to see how many you had?
Ophrah	Yes.
Mrs. Porter	I saw a lot of people doing that. And that's fine because counting by ten is something a lot of you are very comfortable doing. But Ophrah already showed one problem is that it's easy to make a mistake. And, there are so many numbers that it takes a long time. It's OK. How many more people counted by tens? I saw quite a few, I thought. You're counting by tens? Did you get 209?

In this exchange with Ophrah, Mrs. Porter revoiced almost line by line again so that she could help her communicate her strategy to other members of the classroom community. Ophrah was not the first student to volunteer to report her strategy but she was called on after three students had already explained their strategies. The evidence that she had made an error positioned Ophrah as a student who was developing important mathematical skills but did not demonstrate advanced proficiency. Mrs. Porter's last utterance shows that there were other students who had used the same strategy. Moreover, Mrs. Porter made use of Ophrah's strategy in order to inform the other students of the possibility of making mistakes. Ophrah was aligned by Mrs. Porter with those who used the same strategy that could lead to calculation errors. Mrs. Porter's message to Ophrah and her classmates was that using strategies that make sense (e.g., adding instead of subtracting when you need to regroup) is more important than using conventional strategies if you don't fully understand them. Thus, we believe that this exchange represents another instance of the reform storyline: accuracy is less important than meaningfulness and where even "good" mathematicians make mistakes.

The next excerpt shows the conversation between Mrs. Porter and Pulak on the same day. Pulak was the first student who was called on that day. Although he did not talk loudly enough to be heard on the audiotape, we could understand what he had said through Mrs. Porter's revoicing. In order to solve the problem, he had rounded 1789 up by adding 1 so that he could operate on 1790. This made it easier to do the calculation because it did not require regrouping. Then he subtracted 1790 from 1998 to get 208 and then remembered to add 1 to his final answer to get 209.

Excerpt 4: Flexibility

Mrs. Porter	OK, Pulak then what did you do?
Pulak	(not audible)
Mrs. Porter	OK, then you did 1998 minus 1790 and when you do that, you have no regrouping to do, right? But you would have had regrouping to do if you left it at 89, so would you do it for us? OK, let me put that down, 9 minus 7 is 2…uhm, 9 minus 9 is zero…8 minus zero is 8… but it wasn't 1790, that was close, so what did you do? Uhm, so you added one year to that… OK, he added one year to that, and he got to 209.

Interestingly, Mrs. Porter's utterance ("when you do that, you have no regrouping to do, right?") implies that Mrs. Porter had expected Pulak to use the standard algorithm, which requires regrouping. However, he invented a different strategy. As a result, he showed that he was capable of not only regrouping, which he had shown in Excerpt (2), but also of inventing a new strategy. In this excerpt, it appears that Pulak was conforming to the reform storyline, since he used an invented strategy to solve this problem.

This excerpt appears to disconfirm our initial hypothesis that Mrs. Porter would never use revoicing with Pulak and that he would always use conventional strategies. As Erickson and Schultz (1981) recommended, once a pattern is identified, then researchers need to systematically search for additional instances that may confirm or disconfirm their hypotheses. Nevertheless, this exchange provides another instance of Pulak's proficiency and also of his flexibility (another aspect of the reform storyline introduced on the first day of class). It also shows another example of Mrs. Porter refusing to explain to the rest of the class her private conversation with Pulak about "regrouping" that both of them obviously understood. Note that most of the pronouns employed by the teacher in this excerpt were "you" (addressed to Pulak as her audience) except for the final pronoun, "he" (addressed to his classmates). Thus, this episode indicates that the reform and conventional storylines may sometimes be combined and that mathematical proficiency could be defined using both storylines, at least in the case of Pulak.

On October 15, Mrs. Porter gave two problems to her students. The second of these two problems was: A group of 252 ghosts were needed to haunt 9 cemeteries. How many ghosts went to each cemetery? Ophrah's and Pulak's solutions to this division problem are depicted below.

Excerpt 5: Broadcasting a strategy

Ophrah	I. . . I drew 9 boxes but there were 2 other boxes under them.
Mrs. Porter	(She apparently didn't hear Ophrah)
Ophrah	I drew a box under them. (She also drew a ghost figure representing the number 5 and a puffy rounded figure representing the number 2.)
Mrs. Porter	She can count faster by 5s and 2s than she can by ones.
Ophrah	On the bottom, I put 5.
Mrs. Porter	(Drawing on the board. There were 9 tomb stone images and she was writing the number 5 in each one repeatedly as Ophrah explained her solution to the problem.) Up to this point, your strategy worked very well. . . So, how could we finish this?
Ophrah	(inaudible)

Mrs. Porter revoiced Ophrah's strategy on this day, but we could see the difference in the pattern of revoicing between this excerpt and the previous two that involved her [i.e., Excerpts (1) and (3)]. In this instance, Mrs. Porter did not revoice verbally or line by line but rather broadcast Ophrah's strategy by drawing it on the white board in the front of the classroom so her classmates could see it. Mrs. Porter's two comments (e.g., "She can count faster by 5s and 2s than she can by one," and "Up to this point your strategy worked very well.") appeared to be designed for their audience (the first comment) and as evaluative feedback to Ophrah (the second comment). The purpose of her revoicing here seemed different than in the previous excerpts. In those lessons, revoicing seemed designed to help Ophrah better communicate her strategies to members of the classroom community. In this lesson, Mrs. Porter appeared to use revoicing in order to provide feedback to Ophrah and also comment on her strategy (as in the conventional discursive pattern of Initiation-Response/Feedback) (Mehan, 1979; Wells, 1993).

In contrast, Pulak solved the same problem by using the conventional long-division algorithm (252÷9=28), and got the correct answer.

Excerpt 6: Competitors

Pulak	252 divided by 9 equals 28. (Mrs. Porter writes his solution on the board as he narrates it.) 9 times 3 was too much, 9 times 2 equals 18. 25 minus 18 equals 7. Bring down 2 to make 72.
Mrs. Porter	In order to do this kind of division, you need to know your multiplication tables well. And again, I don't expect third graders to know how to do this…
Ophrah	He always does stuff like that. (Mumbled to the students at her table.)

This excerpt is interesting for two reasons. First, Mrs. Porter did not revoice to broadcast Pulak's solution verbally as she wrote and did not explain it to his classmates. She just redid what he had done in order to show his approach to division to the other students. After he was finished narrating his strategy, she added, "In order to do this kind of division, you need to know your multiplication tables well" and "I don't expect third graders to know how to do this." Again, her utterances positioned Pulak as an advanced math student who is able to do things that only older students can do. In response, Ophrah quietly reflected on Pulak's proficiency to a few of her classmates: "He always does stuff like that." This comment appeared to reflect her jealousy toward Pulak. Thus, Ophrah seemed to position herself as a competitor and positioned Pulak as someone with whom she (unsuccessfully) competed.

The first problem earlier in that same day also required division and Pulak had used the conventional algorithm to solve that problem as well. Again, on this occasion, Mrs. Porter repeated what Pulak had done rather than helping him explain his strategy to his classmates. She told him to show the division and wrote it down on the board. Pulak successfully did the division and got the correct answer. Moreover, he did multiplication to check his strategy.

Excerpt 7: Advanced mathematics student

Mrs. Porter	OK. Pulak.
Pulak	I did 51 minus 3 and I did 48 divided by 3.
Mrs. Porter	Pulak come show us what you did. (Pulak goes up to the board. Mrs. Porter writes as he talks.)
Pulak	1 times 3 equals 3. Four minus 3 equals 1. Drop down the 8.
Mrs. Porter	3 times what is 18?
Pulak	6.
Mrs. Porter	This is something you don't do in third grade.
Pulak	And I did 16 times 3 equals 48.
Mrs. Porter	And he checked his work. . . He used 2 algorithms he has learned. And that worked very well for him.

Mrs. Porter positioned Pulak again as an advanced mathematics student. One of her utterances ("This is something you don't do in third grade.") may have had a significant impact on Pulak's position in the class. Pulak was separated again from his peers by the fact that he was able to use the division algorithm. Although Pulak refrained from positioning himself, Mrs. Porter positioned him as possessing a secret that only older students know. Also, as it had happened in September, Nathan expressed his desire to understand and use the division algorithm. Instead of complying with Nathan's repeated requests, Mrs. Porter gave him an example of why he should use invented procedures not the algorithm to solve this problem. The beginning of their exchange appears below:

Nathan	I don't understand what Pulak did.
Mrs. Porter	That's OK. You'll learn more of it.

Nathan	But can we learn it now?

Once again we see the intrusion of the conventional storyline without the usual instructional support that would enable Pulak's classmates (such as Nathan) to learn to master this secret (sophisticated) strategy.

Although Mrs. Porter kept discouraging the use of conventional algorithms by students who didn't understand them, we found students trying to use them throughout our four months of observations. This situation reappeared in the middle of December, when Mrs. Porter asked her students to solve a measurement conversion problem that is conventionally solved using multiplication: The Eiffel Tower is 984 feet high. How many inches would that be? Pulak was absent on that day, so we had a chance to observe what occurred without his presence.

The first student to report her strategy, Lyndsey, used repeated addition to solve the problem (i.e., adding 984 twelve times). After Mrs. Porter announced that Lyndsey's strategy "made sense" to her, she asked if someone had solved the problem a different way. The next volunteer was Ophrah who had used two different strategies: first, she tried to use the conventional multiplication algorithm (starting by multiplying 984 by 2), but when she was not sure how to proceed with that algorithm, she switched to Lyndsey's strategy. During the exchange between Ophrah and her teacher, Mrs. Porter revoiced Ophrah repeatedly.

The rest of the students used a variety of approaches to solve the problem, with almost one-third of the class (Ophrah, Raj, Karl, Miranda, and Nathan) trying, unsuccessfully, to use a conventional multiplication algorithm that each said they had been taught at home (by their father or mother). Later in the class, Mrs. Porter helped two students, Nathan and Ophrah, combine their strategies to solve the problem in this form: (984 × 2) + (984 × 10). This exchange is displayed in excerpt 8.

Excerpt 8: A mathematical resolution

Ophrah	Ahh, no. Uhm, I did it like . . .
Mrs. Porter	You did it like this, 984 <u>times 2</u> and you said,
Ophrah	<u>times 2</u>[iv]
Mrs. Porter	And you said
Ophrah	I said, 2 times 4 is 8, uhm, 2 times 6 is 12.
Mrs. Porter	There's no 6. There's no 6.
Ophrah	I mean 2 times 8 is 16, carry your one. Uhm, 2 times 9 is 18 plus 1 is 19.
Mrs. Porter	Now, she has 984 plus 2 equals, and then she's got 1968 (spoken 19-68). Now, what can she do with this now?
Nathan	Oohhhh, oohhhh. [Nathan is waving his hand so rapidly he falls off of his seat.]
Mrs. Porter	Nathan.
Students	(giggle)
Nathan	Just add a zero, just add a zero, cause it's (unclear).
Mrs. Porter	Does she want to add, does she want 10 of these? Or what does she add a zero to?

Nathan	To this, she wants, she wants just one of those. [It isn't obvious whether he means 10 times 1968 or 10 times 984.]
Students	oohhooo.
Mrs. Porter	I'm not sure about that.
Nathan	Because . . .
Mrs. Porter	Because, if you add a zero it's like. . .
Nathan	It's the magic of 10 because you took away the two.
Mrs. Porter	OK. But what do we want 10 of? Do we want ten of these?
Nathan	No, ya.
Mrs. Porter	I don't think so. What do we want 10 of?
Michael	Ten of 984.
Mrs. Porter	Right. We want ten of these.
Nathan	Yah. That's what I meant.
Mrs. Porter	So, we could say, we've got how many is, we have said that 2 times 984 is 1968. Now 10 more of them. And if you know the magic of ten we just take nine-hundred and eighty-four. . ., (she chuckles), 984 and put a zero. And then you can just add these two numbers together. And look how fast THAT is!

In this exchange, Mrs. Porter began by referring back to Ophrah's first strategy, "Let's look at what Ophrah's decided she was going to do. If she did 984 times 2, did you add them together to get the answer Ophrah?" She positioned Ophrah as someone decisive (at least initially), with a plan for solving the problem that is promising from the teacher's perspective. By revoicing and correcting, Mrs. Porter broadcast Ophrah's use of the conventional multiplication algorithm to her classmates ("There's no 6."). Ophrah promptly corrected herself by saying "I mean 2 times 8 is 16, carry your one." At this point, Ophrah seemed unable to answer her teacher's question, which was addressed to her classmates, "Now, what can she do with this now?" Then, Nathan (and another classmate, Michael) joined in with a strategy for computing 984 X 10 to finish the problem. Nathan, who had been waving his hand so rapidly that he fell off his seat, reflected the excitement of the rest of the class who exclaimed when the solution was finally revealed. After Mrs. Porter recognized Nathan, he suggested adding zero to one of the numbers being considered, as the result of multiplying by the "magic of 10." Because it was not entirely clear to Mrs. Porter (and perhaps most of the class) which number (1968 or 984) Nathan meant, Michael provided that number when he responded to the teacher's request for clarification ("What do you want 10 of?"). Nathan positioned himself as aligned with Michael's answer of 984 when he said, "That's what I meant." At the end of this exchange, Mrs. Porter wrote the following on the board and said "And then you can just add these two numbers together. And look how fast THAT is!"

$$\begin{aligned} 984 \times 2 &= 1968 \\ 984 \times 10 &= \underline{9840} \\ &\ 11808 \end{aligned}$$

After this excerpt ended, Mrs. Porter cautioned her students to avoid conventional algorithms unless they understand how and why they work. Thus, she reinforced the reform storyline, despite the fact that both storylines (the conventional and the reform) played parts in this excerpt. In addition, unlike the earlier excerpts, several students and the teacher produced a collective invented solution to this problem. Mrs. Porter positioned herself with her students several times in this excerpt when she used “we”: “But what do we want 10 of?”; “We want ten of these.”; “So, we could say.” Unlike her earlier exchanges with Pulak, in which she positioned him with her and other adults in using conventional algorithm, here she positioned her students with her when they used an invented strategy to solve this problem.

DISCUSSION AND IMPLICATIONS FOR EDUCATION

What do our analyses of positioning tell us about the formation of mathematical identities in this third grade classroom community? First, we will review the conversational exchanges between Mrs. Porter and Ophrah and Pulak in terms of their potential impact on each student’s identity formation. We will also discuss the importance of storylines in these exchanges. Second, we will address the implications of this study for research methods in investigations of identity. Finally, the implications of this study for research on teaching and teacher education will be outlined.

Implications of positioning for students’ identity formation.

It is clear that Mrs. Porter revoiced Ophrah and Pulak differently and she used revoicing for different purposes. She usually revoiced Ophrah line by line in order to help communicate her strategies to other members of the classroom community. We found several instances of warranted inferences during the discussions between Mrs. Porter and Ophrah. This shows that Mrs. Porter attempted to create a participant framework, in which students have more agency and power than in the IRE/F (Mehan, 1979; Wells, 1993). As a result, Mrs. Porter reflectively positioned herself as someone interested in sharing intellectual authority with her students, like Ophrah. In addition, the teacher’s frequent revoicing positioned Ophrah as an active member of the classroom community. In Mrs. Porter’s classroom community, using invented strategies that make sense to students was highly valued over using conventional strategies without understanding. Ophrah did not rely on conventional algorithms but frequently used information from the multiplication tables that made sense to her. She used this multiplication table strategy quite frequently for problem-solving, although she did not always succeed. Even when Ophrah failed to get an accurate answer to a problem, Mrs. Porter commented on her courage, persistence, and flexibility, qualities that were valued within the reform storyline for mathematical proficiency. Also, when Ophrah made mistakes, Mrs. Porter mentioned that other members of the classroom community had made similar errors and she often referred to Ophrah’s strategies as logical. Thus, Ophrah was positioned by her teacher as conforming to the norms of mathematical proficiency in the reform storyline.

Mrs. Porter interacted quite differently with Pulak. She rarely revoiced his strategies to broadcast them to his classmates. During the first week of school, she mentioned Pulak's brother's mathematical proficiency and positioned Pulak as an advanced student in terms of his likely familiarity with the classroom social norms and with the mathematics they would be studying. From very early in the school year, Pulak was privileged as someone likely to display advanced mathematical proficiency. Through her explicit messages about his proficiency (e.g., "this is something you don't do in third grade"), her tendency to avoid revoicing his explanations and to address her comments to him and not to his classmates, Mrs. Porter effectively positioned Pulak outside the classroom community and instead inside a larger community of older children and adults (e.g., "That's the way a lot of your parents would do it"). In this way, she positioned Pulak as an advanced mathematics student, despite his periods of nonconformity to the reform storyline. We have limited evidence that Pulak reflectively positioned himself as highly proficient in mathematics, although he did write that mathematics was his favorite subject.

The tri-polar structure of conversations in positioning theory gives a significant role to storylines produced during conversations. In the excerpts reviewed previously, Mrs. Porter tried to maintain a reform mathematics storyline, characterized by frequent mathematical discussions and active student participation in sense-making activities. Our analyses of alignments through revoicing enabled us to see Mrs. Porter and her students as they engaged in positioning (both interactional and reflective). It also helped us understand how two students (Ophrah and Pulak) formed identities as math thinkers through such positioning. Ophrah's identity as a math thinker changed (from average to above average proficiency), at least for the teacher, after she repeatedly positioned herself and was positioned as a student who conformed to the reform storyline. Pulak's identity as a math thinker did not change during the course of our study. He was consistently positioned by Mrs. Porter as an advanced math student. The fact that Pulak used strategies Mrs. Porter did not expect other students to use sometimes positioned him as the one who did not conform to a reform storyline. However, this positioning simultaneously located him in a privileged position in the class. In both cases, we would argue that positioning made important contributions to the mathematical identities of these two students.

We agree with scholars who are aware of the significant role of positioning in identity formation. Bucholtz and Hall (2005), for example, defined identity as the "social positioning of self and other" (p. 586) and proposed positionality as one of the fundamental principles for identity research. Our results are consistent with Gee's (2001) reference to perceptions of others in defining identity (i.e., his definition of identity as being recognized as a certain kind of person). How one is viewed determines his or her identity, and identity is temporary, changeable, and unstable in nature. Thus, concepts of position and positioning have become critical to identity research because of a movement within discursive psychology toward dynamic theories and methods and away from static and essentialist approaches.

Implications for research methods

Studies of identity have often relied on interviews (e.g., Boaler & Greeno, 2000; Sfard & Prusak, 2005). This method allows one to examine the personal narratives of adolescents and adults and relate them to identity formation. Unfortunately, preadolescent children are less self-aware and articulate about their lives and so interviews may not be the preferred method for assessing identity formation at younger ages. In addition, theories of learning, such as that of Lave and Wenger (1991) require methods capable of examining the norms and practices through which learning (and identity formation) are said to occur. Thus, we need theoretical frameworks, like positioning, that can guide ethnographic studies of interactive practices in communities.

Ethnographic studies require a logic of inquiry (Green et al., 2003) so that crucial decisions about research design, data collection, sampling, indexing, representation, and interpretation can be articulated, justified, replicated, and triangulated. One important way to triangulate classroom ethnography is by member-checking: having key participants (e.g., the teacher) review preliminary findings. Another way to triangulate is to collect different types of materials so that one can systematically search for data that confirm and disconfirm evolving hypotheses. Convergent findings across different data sources, different informants, different occasions and settings increase the reliability and validity of the study. Because of the wealth of data that is typically collected in an ethnographic study, a limited number of cases can be investigated. Multiple cases allow one to engage in the comparative analysis required by a logic of inquiry and should be informed by a replication logic (Yin, 2003).

Implications for teacher education and research on teaching

Positioning locates people in a particular conversational space. During conversations participants always utter from a certain perspective, and their discursive locations reflect their own point of view. When teachers and students talk in a classroom, all their utterances are generated from a particular position or perspective. Mrs. Porter espoused the instructional goals of mathematics reform and, as a result, she often positioned her students in accordance with the reform storyline. Whether consciously or not, teachers have temporary control over the social positions of their students, and their power influences both the dynamics of and social relations within a classroom community. As classrooms become more student-centered and communication-rich, teachers need to be aware of their inevitable positioning and its long-term influence on their students. Mrs. Porter was quite articulate about the long-term influence of the conventional storyline in mathematics education on her own identity as a learner and we believe its effect could be seen in her classroom many years later (Forman & Ansell, 2001).

After she reviewed a draft of an earlier write-up on her classroom, Mrs. Porter defended her decision to discourage many of her students from using the conventional algorithms to solve problems. She argued that most students begin third grade with an inadequate conceptual understanding of place value. Thus, introducing the algorithms that are based on place value too early can merely confuse and frustrate children. She wrote to us that "I truly believe that a focus on traditional algorithms can be HARMFUL to children" (emphasis in the original commentary). Instead, she advocated encouraging them to use other strategies (e.g., addition instead of subtraction or repeated addition instead of multiplication) while they are consolidating their understanding of place value.

Mrs. Porter seemed to be very aware of the challenges facing teachers when they try to help students make sense of difficult mathematical operations despite the messages from older family members that may encourage them to memorize number facts and mathematical procedures without understanding why they work (Forman & Ansell, 2001). To complicate this dilemma even further, we are aware of another set of challenges facing teachers who are attempting to change the discourse practices in their classrooms: how positioning of students can affect their identities as competent mathematics students. When traditional classroom discourse patterns (e.g., Initiation, Response, Evaluation or I-R-E) are altered, then teachers may be positioning some students as advanced and others as average or struggling, merely by their choice of words or phrases. Seemingly innocuous comments such as "that is logical but not very efficient" or "that's not something third graders know" may privilege some students and silence others (Forman & Ansell, 2002). In addition, students' invented strategies may be difficult to interpret or compare, without prior preparation and/or many years of experience with reform teaching (Forman, McCormick, & Donato, 1998). Finally, teachers will need to create norms, like those in Mrs. Porter's classroom that create a caring community where students know that they can take risks without being unfairly judged as lacking in ability (Forman & Ansell, 2005; Hatano & Inagaki, 1998; Yackel & Cobb, 1996).

Generally speaking, teachers have more power than students in classrooms. This is true even if teachers create a learning environment in which students have a considerable degree of agency. Mrs. Porter's classroom was reform-oriented, that is, she allowed the students to take the floor more often, share ideas, and learn from each other. However, it was ironic that her focus on a reform storyline limited some students' access to particular forms of privileged knowledge. She did not encourage her students (except Pulak and Raj) to use or understand conventional algorithms, despite repeated requests by students such as Nathan. She ended up privileging the student or students who could be successful using both invented and conventional strategies by positioning them as members of the adult community outside of the classroom. Thus, teachers may need to modify their reform agenda to include practices and norms that might be quite adaptive for students when they leave their current classroom community and try to join other mathematical communities (at home, in their neighborhoods and at school).

i An earlier version of this chapter was presented as part of a poster symposium, "Dynamics of positioning: Perspectives on students' participation in relation to each other, academic disciplines, and classroom settings" (Randi Engle, Chair) at the Annual Meeting of the American Educational Research Association, Chicago, IL, April 2007. The research reported herein was supported, in part, by grants to the second author from the Spencer Foundation, from the U.S. Department of Education, Office of Educational Research and Improvement, to the National Center for Improving Student Learning and Achievement in Mathematics and Science (R305A60007-98), and from the School of Education at the University of Pittsburgh. The opinions expressed do not necessarily reflect the position, policy, or endorsement of the supporting agencies. The authors gratefully acknowledge the assistance of Renee Bruckner, Deborah Dobransky-Fasiska, Amanda Godley, Jaime Munoz, and Elaine Olds.

ii There are two different kinds of alignments involved in arguments: aligning (or positioning) students versus explanations. Sometimes these alignments agree (a group of students affiliate with each other AND agree with each other) but sometimes they do not agree (a group of students affiliate with each other BUT disagree with each other).

iii The teacher's and students' names are all pseudonyms.

iv Overlapping speech between two speakers is indicated by underlines.

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